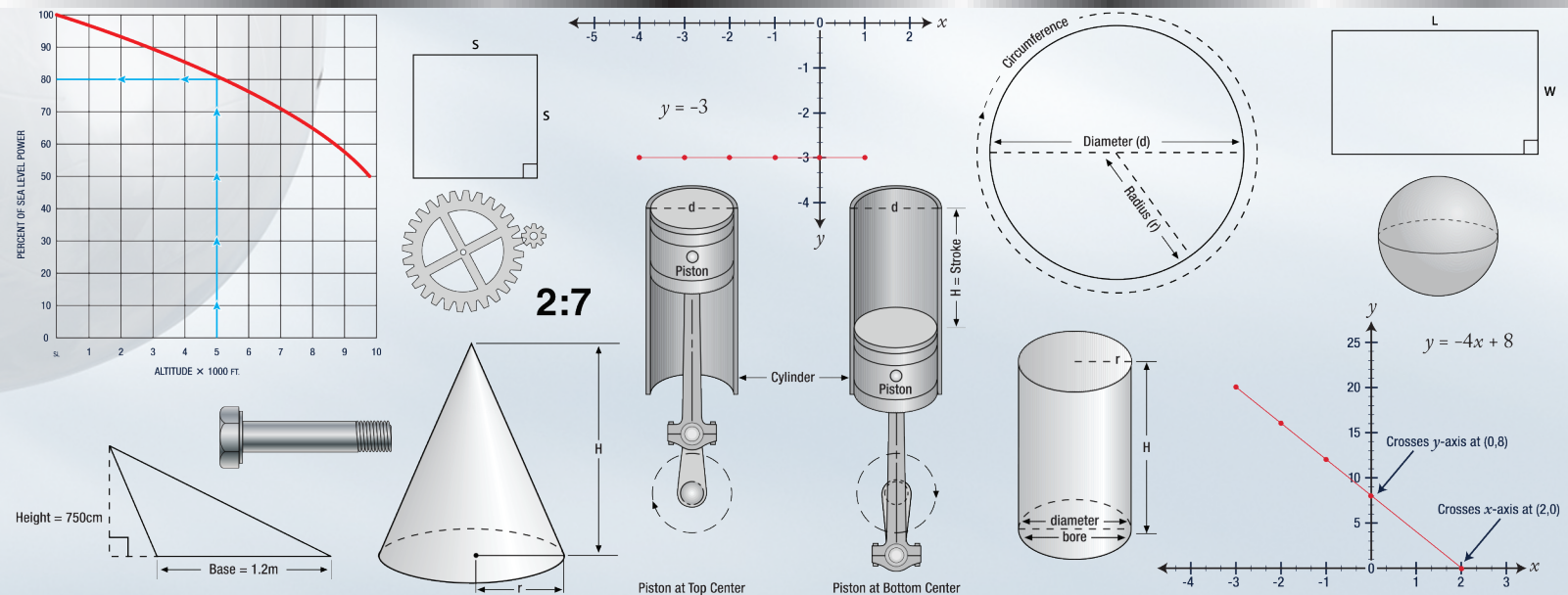


MATHEMATICS

Aviation Maintenance Technician Certification Series



- Arithmetic
- Algebra
- Geometry

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MODULE 01

FOR B1 & B2 CERTIFICATION

MATHEMATICS

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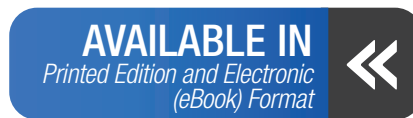
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AVIATION MAINTENANCE TECHNICIAN CERTIFICATION SERIES

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WELCOME

The publishers of this Aviation Maintenance Technician Certification Series welcome you to the world of aviation maintenance. As you move towards EASA certification, you are required to gain suitable knowledge and experience in your chosen area. Qualification on basic subjects for each aircraft maintenance license category or subcategory is accomplished in accordance with the following matrix. Where applicable, subjects are indicated by an "X" in the column below the license heading.

For other educational tools created to prepare candidates for licensure, contact Aircraft Technical Book Company.

We wish you good luck and success in your studies and in your aviation career!

REVISION LOG

VERSION	EFFECTIVE DATE	DESCRIPTION OF CHANGE
001	2016 01	Module Creation and Release
002	2017 03	Format Update and Minor Corrections

FORWARD

PART-66 and the Acceptable Means of Compliance (AMC) and Guidance Material (GM) of the European Aviation Safety Agency (EASA) Regulation (EC) No. 1321/2014, Appendix 1 to the Implementing Rules establishes the Basic Knowledge Requirements for those seeking an aircraft maintenance license. The information in this Module of the Aviation Maintenance Technical Certification Series published by the Aircraft Technical Book Company meets or exceeds the breadth and depth of knowledge subject matter referenced in Appendix 1 of the Implementing Rules. However, the order of the material presented is at the discretion of the editor in an effort to convey the required knowledge in the most sequential and comprehensible manner. Knowledge levels required for Category A1, B1, B2, and B3 aircraft maintenance licenses remain unchanged from those listed in Appendix 1 Basic Knowledge Requirements. Tables from Appendix 1 Basic Knowledge Requirements are reproduced at the beginning of each module in the series and again at the beginning of each Sub-Module.

How numbers are written in this book:

This book uses the International Civil Aviation Organization (ICAO) standard of writing numbers. This method displays large numbers by adding a space between each group of 3 digits. This is opposed to the American method which uses commas and the European method which uses periods. For example, the number one million is expressed as so:

ICAO Standard	1 000 000
European Standard	1.000.000
American Standard	1,000,000

SI Units:

The International System of Units (SI) developed and maintained by the General Conference of Weights and Measures (CGPM) shall be used as the standard system of units of measurement for all aspects of international civil aviation air and ground operations.

Prefixes:

The prefixes and symbols listed in the table below shall be used to form names and symbols of the decimal multiples and submultiples of International System of Units (SI) units.

MULTIPLICATION FACTOR	PREFIX	SYMBOL
1 000 000 000 000 000 000 = 10^{18}	exa	E
1 000 000 000 000 000 = 10^{15}	peta	P
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^9	giga	G
1 000 000 = 10^6	mega	M
1 000 = 10^3	kilo	k
100 = 10^2	hecto	h
10 = 10^1	deca	da
0.1 = 10^{-1}	deci	d
0.01 = 10^{-2}	centi	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p
0.000 000 000 000 001 = 10^{-15}	femto	f
0.000 000 000 000 000 001 = 10^{-18}	atto	a

International System of Units (SI) Prefixes

EASA LICENSE CATEGORY CHART

Module Number and Title		A1 Airplane Turbine	B1.1 Airplane Turbine	B1.2 Airplane Piston	B1.3 Helicopter Turbine	B2 Avionics
1	Mathematics	X	X	X	X	X
2	Physics	X	X	X	X	X
3	Electrical Fundamentals	X	X	X	X	X
4	Electronic Fundamentals		X	X	X	X
5	Digital Techniques / Electronic Instrument Systems	X	X	X	X	X
6	Materials and Hardware	X	X	X	X	X
7A	Maintenance Practices	X	X	X	X	X
8	Basic Aerodynamics	X	X	X	X	X
9A	Human Factors	X	X	X	X	X
10	Aviation Legislation	X	X	X	X	X
11A	Turbine Aeroplane Aerodynamics, Structures and Systems	X	X			
11B	Piston Aeroplane Aerodynamics, Structures and Systems			X		
12	Helicopter Aerodynamics, Structures and Systems				X	
13	Aircraft Aerodynamics, Structures and Systems					X
14	Propulsion					X
15	Gas Turbine Engine	X	X		X	
16	Piston Engine			X		
17A	Propeller	X	X	X		

MODULE 01 SYLLABUS AS OUTLINED IN PART-66, APPENDIX 1.

LEVELS

CERTIFICATION CATEGORY →

B1 B2

Sub-Module 01 - Arithmetic

Arithmetical terms and signs, methods of multiplication and division, fractions and decimals, factors and multiples, weights, measures and conversion factors, ratio and proportion, averages and percentages, areas and volumes, squares, cubes, square and cube roots.

2

2

Sub-Module 02 - Algebra

(a) Evaluating simple algebraic expressions, addition, subtraction, multiplication and division, use of brackets, simple algebraic fractions;

2

2

(b) Linear equations and their solutions;
Indices and powers, negative and fractional indices;
Binary and other applicable numbering systems;
Simultaneous equations and second degree equations with one unknown;
Logarithms;

1

1

CERTIFICATION CATEGORY →		LEVELS	
		B1	B2
Sub-Module 03 - Geometry			
(a)	Simple geometrical constructions	1	1
(b)	Graphical representation; nature and uses of graphs, graphs of equations/functions;	2	2
(c)	Simple trigonometry; trigonometrical relationships, use of tables and rectangular and polar coordinates.	2	2

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PART-66 SYLLABUS LEVELS
CERTIFICATION CATEGORY →

B1	B2
2	2

Sub-Module 01
ARITHMETIC

Knowledge Requirements

1.1 - Arithmetic

Arithmetical terms and signs, methods of multiplication and division, fractions and decimals, factors and multiples, weights, measures and conversion factors, ratio and proportion, averages and percentages, areas and volumes, squares, cubes, square and cube roots.

Level 2
A general knowledge of the theoretical and practical aspects of the subject and an ability to apply that knowledge.

- Objectives:*
- (a) The applicant should be able to understand the theoretical fundamentals of the subject.
 - (b) The applicant should be able to give a general description of the subject using, as appropriate, typical examples.
 - (c) The applicant should be able to use mathematical formula in conjunction with physical laws describing the subject.
 - (d) The applicant should be able to read and understand sketches, drawings and schematics describing the subject.
 - (e) The applicant should be able to apply his knowledge in a practical manner using detailed procedures.

ARITHMETIC IN AVIATION MAINTENANCE

Arithmetic is the branch of mathematics dealing with the properties and manipulation of numbers. Performing arithmetic calculations with success requires an understanding of the correct methods and procedures. Arithmetic may be thought of as a set of tools. The aviation maintenance professional will need these tools to successfully complete the maintenance, repair, installation, or certification of aircraft equipment.

Arithmetic is the basis for all aspects of mathematics. Math is used in measuring and calculating serviceability of close tolerance engine components, when calculating

the weight and balance for the installation of new avionics and more. A sound knowledge and manipulation of mathematic principles is used on a regular basis during nearly all aspects of aircraft maintenance.

The level to which an aviation maintenance student is required to have knowledge of arithmetic is listed on *page 1.1*, according to the certification being sought. A description of the applicable knowledge levels are presented and will be included at the beginning of each *sub-module* in the module.

ARITHMETIC

In this *sub-module*, arithmetic terms, positive, negative and denominated numbers, and fractions and decimals are explained. The use of ratios and proportions, calculation of averages and percentages and the calculation of squares, cubes, and roots are also explained.

THE INTEGERS

Integers are all positive and negative whole numbers including 0.

WHOLE NUMBERS

Whole numbers are the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13... and so on.

Digits are the whole numbers between 0 and 9.

When a large whole number is written, each digit within the number has a specific place and cannot be moved. This is known as *place value*. Each digit occupies a place within the whole number which determines the value of the entire number and must be maintained.

Example:

The number 285 contains 3 digits: 2, 8 and 5. The 2 in the position shown represents 200, the 8 in the position shown represents 80 and, the 5 in the position shown represents 5. Thus, the number 285.

If the digit positions are changed, say 852, the number is not the same. Each digit now has a different place within the number and the place value has changed. Now the 8 represents 800, the 5 represents 50 and the 2 represents 2.

Therefore, always maintain the digits in the order they are given within a number, and whenever adding subtracting, multiplying, or dividing numbers, be sure to maintain the place value of the digits. **Figure 1-1** illustrates the place value of digits within whole number.

PLACE VALUES					
	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES
35 →				3	5
269 →			2	6	9
285 →			2	8	5
852 →			8	5	2
12 749 →	1	2	7	4	9

Figure 1-1. Place Values.

ADDITION OF WHOLE NUMBERS

Addition is the process in which the value of one number is added to the value of another. The result is called the *sum*.

$$\begin{array}{r} 2 + 5 = 7 \text{ or, } 2 \\ + 5 \\ \hline 7 \end{array}$$

When adding several whole numbers, such as 4 314, 122, 93 132, and 10, align them into columns according to place value and then add.

$$\begin{array}{r} 4\,314 \\ 122 \\ 93\,132 \\ + 10 \\ \hline 97\,578 \end{array}$$

← This is the *sum* of the four whole numbers.

If the sum in a column is greater than 9, write the right digit at the bottom of the column and add the left digit into the sum of the column immediately to the left.

$$\begin{array}{r} 1 \\ 234 \\ 708 \\ 40 \\ \hline 982 \end{array}$$

← The sum of the right-most column ($4 + 8 + 0$) is equal to 12. So the 2 is put in the right most column total space and the 1 is added into the sum of the column immediately to the left. ($1 + 3 + 0 + 4 = 8$)

SUBTRACTION OF WHOLE NUMBERS

Subtraction is the process in which the value of one number is taken from the value of another. The answer is called the *difference*. When subtracting two whole numbers, such as 3 461 from 97 564, align them into columns according to place value and then subtract.

$$\begin{array}{r} 97\,564 \\ -3\,461 \\ \hline 94\,103 \end{array}$$

← This is the *difference* of the two whole numbers.

When subtracting in any column, if the digit above the digit to be subtracted is smaller than the digit to be subtracted, the digit directly to the left of this smaller number can be reduced by 1 and the small digit can be increased by 10 before subtraction takes place.

$$\begin{array}{r} 31 \\ -7 \\ \hline 24 \end{array} \quad \rightarrow \quad \begin{array}{r} 2 \\ -0 \\ \hline 2 \end{array} \quad \begin{array}{r} 11 \\ -7 \\ \hline 4 \end{array}$$

MULTIPLICATION OF WHOLE NUMBERS

Multiplication is the process of repeated addition. The result is called the *product*. For example, 4×3 is the same as $4 + 4 + 4$.

Example:

How many hydraulic system filters are in the supply room if there are 5 cartons and each carton contains 4 filters? By repeated addition:

$$4 + 4 + 4 + 4 + 4 = 20 \quad \text{or} \quad 4 \times 5 = 20$$

Multiplication of large numbers involves the memorization of all combinations of the multiplication of digits. **Figure 1-2** illustrates the products of all digits when multiplied.

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Figure 1-2. A multiplication table that includes the products of all digits.

When multiplying large numbers, arrange the numbers to be multiplied vertically. Be sure to align the numbers in columns to protect the place value of each number.

Beginning with the right-most digit of the bottom number, multiply it by the digits in the top number from right to left. Record each product in the proper place value column. If a product is greater than 9, record the right most digit of the number in the proper place value column and add the left digit into the product of the bottom digit and the next digit to the left in the top number.

Then multiply the next right-most digit in the bottom number by the digits in the top number and record the products in the proper place value columns. Once all digits from the bottom number have been multiplied by all of the top digits, add all of the products recorded to obtain the final answer.

$$\begin{array}{r}
 18 \\
 \times 35 \\
 \hline
 90 \\
 54 \\
 \hline
 630
 \end{array}
 \leftarrow \text{This is the product of } 18 \times 35.$$

When solving this problem, the first step is to multiply the 5 in the number 35 by both digits in the number 18.

The second step is to multiply the 3 in the number 35 by both digits in the number 18. By doing this, both place value holders in the number 35 are multiplied by both place value holders in the number 18. Then, the result from steps 1 and 2 are added together. Note that recording of the products in each step must begin directly under the digit being multiplied on the second line and proceeds to the left.

In the following example, the same procedure is used. Each place value holder in the numbers to be multiplied are multiplied by each other.

$$\begin{array}{r}
 321 \\
 \times 43 \\
 \hline
 963 \\
 1284 \\
 \hline
 13803
 \end{array}
 \leftarrow \text{This is the product of } 43 \times 321.$$

DIVISION OF WHOLE NUMBERS

Division is the process of finding how many times one number (called the *divisor*) is contained in another number (called the *dividend*). The result is the *quotient*, and any amount left over is called the *remainder*. A division problem may be written as follows:

$$\begin{array}{c}
 \text{quotient} \\
 \hline
 \text{divisor} \overline{) \text{dividend}}
 \end{array}
 \quad \text{or} \quad
 \frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

or

$$\text{dividend} \div \text{divisor} = \text{quotient}$$

Using numbers, say 12 divided by 3, the problem could look like this:

$$3 \overline{) 12} \quad \text{or} \quad \frac{12}{3} \quad \text{or} \quad 12 \div 3$$

When dividing small numbers, the multiplication table shown in **Figure 1-2** can be used. Think of any of the numbers at the intersection of a column and row as a dividend. The numbers at the top and the left side of the table column and row that intersect are the divisor and the quotient. A process known as long division can also be used to calculate division that involves large numbers. It is shown in the following example.

Long division involves repeated division of the divisor into place value holders of the dividend from left to right.

Example:

239 landing gear bolts need to be divided between 7 aircraft. How many bolts will each aircraft receive?

$$\begin{array}{r}
 34 \\
 7 \overline{) 239} \\
 \underline{-21} \\
 29 \\
 \underline{-28} \\
 1
 \end{array}
 \begin{array}{l}
 \leftarrow \text{quotient} \\
 \leftarrow \text{dividend} \\
 \leftarrow \text{remainder}
 \end{array}$$

Note that the divisor is greater than the left most place value holder of the dividend. The two left most place value holders are used in the first step of repeated division.

When the divisor is greater than a single digit number, the number of left most place value holders in the dividend used to begin the repeated division process is adjusted until the divisor is smaller. In the following example, the 3 left most place value holders are used.

$$\begin{array}{r}
 73 \\
 18 \overline{) 1325} \\
 \underline{-126} \\
 65 \\
 \underline{-54} \\
 11
 \end{array}$$

FACTORS AND MULTIPLES

Any number that is exactly divisible by a given number is a *multiple* of the given number.

For example, 24 is a multiple of 2, 3, 4, 6, 8, and 12, since it is divisible by each of these numbers. Saying that 24 is a multiple of 3, for instance, is equivalent to saying that 3 multiplied by some whole number will give 24. Any number is a multiple of itself and also of 1.

Any number that is a multiple of 2 is an *even number*. The even numbers begin with 2 and progress by 2's. Any number that is not a multiple of 2 is an *odd number*. Any number that can be divided into a given number without a remainder is a *factor* of the given number. The given number is a multiple of any number that is one of its factors. For example, 2, 3, 4, 6, 8, and 12 are factors of 24. The following equalities show various combinations of the factors of 24:

$$\begin{array}{lll} 1 \times 24 = 24 & 2 \times 12 = 24 & 3 \times 8 = 24 \\ 4 \times 6 = 24 & 6 \times 4 = 24 & 8 \times 3 = 24 \\ 12 \times 2 = 24 & 24 \times 1 = 24 & \end{array}$$

All of these factors of 24 can be found in the multiplication table in *Figure 1-2*, except 12.

LOWEST COMMON MULTIPLE (LCM) AND HIGHEST COMMON FACTOR (HCF)

The lowest common multiple of two or more whole numbers is the smallest positive number which is a common multiple of the numbers.

The easiest way to find the LCM of two or more numbers is to list the multiples and pick out the smallest common multiple.

Example:

The multiples of 4 are 4, 8, 12, 16, 20, 24, ..

The multiples of 6 are 6, 12, 18, 24...

Looking at both lists gives us 12 as the lowest common multiple.

Example:

The LCM of 7 and 8 is 56.

This means if we listed all the multiples of both 7 and 8, 56 would be the smallest common number.

The highest common factor of two or more numbers is the highest number that will divide in to the numbers.

The easiest way to find the highest common factor of two or more numbers is to list all the factors and read off the highest value.

Example:

The factors of 12 are 1, 2, 3, 4, 6, 12.

The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

From the list the highest common factor is 12.

Using lists as shown above to find the LCM and HCF of numbers can be very laborious, even for relatively small numbers. An easier way is to use Prime Factors.

First we need to know what prime numbers and prime factors are.

PRIME NUMBERS

A prime number is a number which has only two factors and those factors are 1 and the number itself.

For example, 7 is a prime number. Its only factors are 1 and 7. 19 is a prime number. Its only factors are 1 and 19. Here is a list of the first eight prime numbers: 2, 3, 5, 7, 11, 13, 17 and 19.

PRIME FACTORS

Every whole number can be written as a product of prime factors.

To write a number as a product of prime numbers, we continuously divide the number by a prime as shown in the following examples.

Example:

Write 924 as a product of prime.

Solution:

$$924 \div 2 = 462$$

$$462 \div 2 = 231$$

Using your knowledge of basic multiplication tables, we know $3 \times 7 = 21$ and since 231 ends in 1 we try dividing by 3 next:

$$231 \div 3 = 77$$

The next step is simple:

$$77 \div 7 = 11$$

We conclude:

$$924 = 2 \times 2 \times 3 \times 7 \times 11$$

Example:

Write 240 as a product of primes.

Solution:

$$240 \div 2 = 120$$

$$120 \div 2 = 60$$

$$60 \div 2 = 30$$

$$30 \div 2 = 15$$

$$15 \div 3 = 5$$

We conclude:

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$

Example:

Here's few more numbers written in terms of prime factors.

$$40 = 2 \times 2 \times 2 \times 5$$

$$231 = 3 \times 7 \times 11$$

$$1\ 638 = 2 \times 3 \times 3 \times 7 \times 13$$

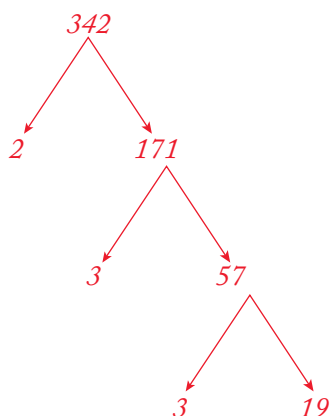
Another way to write numbers in terms of prime factors is shown in the following example.

Example:

Write 342 as a product of primes.

Solution:

Start by dividing the number by its lowest prime factor. In this case the lowest prime factor is 2. Then continue dividing by 3, 5, 7 etc until the only remainder is a prime number.



Write the number as a product of primes.

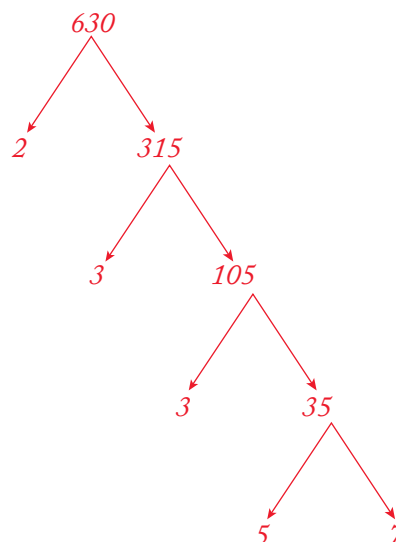
$$342 = 2 \times 3 \times 3 \times 19$$

Example:

Write 630 as a product of primes.

Solution:

Start by dividing the number by its lowest prime factor. In this case the lowest prime factor is 2. Then continue dividing by 3, 5, 7 etc until the only remainder is a prime number.



Write the number as a product of primes.

$$630 = 2 \times 3 \times 3 \times 5 \times 7$$

LOWEST COMMON MULTIPLE USING PRIME FACTORS METHOD

To find the LCM of two or more whole numbers:

1. Write the numbers in terms of prime factors.
2. Multiply each factor the greatest number of times it occurs in either number.

Example:

Find the LCM of 12 and 80.

Solution:

List the prime factors of each.

$$12 = 2 \times 2 \times 3$$

$$80 = 2 \times 2 \times 2 \times 2 \times 5$$

Multiply each factor the greatest number of times it occurs in either number.

12 has one 3, and 80 has four 2's and one 5, so we multiply 2 four times, 3 once, and five once.

$$LCM = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$

This gives us 240, the smallest number that can be divided by both 12 and 80.

Example:

Find the LCM of 9, 15 and 40

Solution:

$$\begin{aligned} 9 &= 3 \times 3 \\ 15 &= 3 \times 5 \\ 40 &= 2 \times 2 \times 2 \times 5 \end{aligned}$$

2 appears at most three times.

3 appears at most twice.

5 appears at most once.

$$LCM = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$$

HIGHEST COMMON FACTOR USING PRIME FACTORS METHOD

1. Write the numbers in terms of prime factors.
2. The product of all common prime factors is the HCF of the given numbers.

Example:

Find the Highest Common Factor (HCF) of 240 and 924.

Solution:

List the prime factors of each number.

$$\begin{aligned} 240 &= 2 \times 2 \times 2 \times 2 \times 3 \times 5 \\ 924 &= 2 \times 2 \times 3 \times 7 \times 11 \end{aligned}$$

The factors that are common to both lists are 2 and 2 and 3.

Therefore the Highest Common Factor of 240 and 924 is:

$$2 \times 2 \times 3 = 12$$

PRECEDENCE

We have introduced whole numbers and seen examples of adding, subtracting, multiplying and dividing whole numbers. These are called operations in mathematics. We will use these operations again in *sub-module 02 - Algebra*.

There is an accepted order in which these operations must be carried out otherwise there could be many answers to one simple calculation. Consider the following example:

Evaluate:

$$8 - 2 \times 4 + 9$$

If we simply carried out the operations as they are written down we would get the answer as follows:

$$\begin{aligned} 8 - 2 &= 6 \\ 6 \times 4 &= 24 \\ 24 + 9 &= 33 \end{aligned}$$

This however is not correct! If you input $8 - 2 \times 4 + 9$ in to an electronic calculator and you will find that:

$$8 - 2 \times 4 + 9 = 9$$

This is correct, of course, because a calculator uses the rules of *precedence*. **Figure 1-3** shows the order in which calculations are carried out.

Priority Order of Calculation	
First	Brackets
Second	Powers and Roots
Third	Multiplication and Division
Fourth	Addition and Subtraction

Figure 1-3. Order in which calculations are carried out table.

Let's look at the example above again, step by step.

$$8 - 2 \times 4 + 9 = 8 - 8 + 9 = 9$$

Here we must multiply before we add or subtract.

The example above could be written in a clearer manner as follows using brackets.

$$8 - (2 \times 4) + 9$$

Here's few more examples:

$$5 + 3 \times 3 = 5 + 9 = 14$$
$$24 \div (4 + 2) = 24 \div 6 = 4$$

$$5 \times 2^2 = 5 \times 4 = 20$$

$$(3 \times 4 - 4)^2 \div 2 = (12 - 4)^2 \div 2 = 8^2 \div 2 = 64 \div 2 = 32$$

If there is more than one set of brackets, we start calculating from the innermost first.

Example:

What is the value of:

$$34 - 2 \times (3 \times (6 - 2) + 3)$$

Solution:

$$\begin{aligned} 34 - 2 \times (3 \times (6 - 2) + 3) \\ = 34 - 2 \times (3 \times (4) + 3) \\ = 34 - 2 \times (3 \times 4 + 3) \\ = 34 - 2 \times (12 + 3) \\ = 34 - 2 \times (15) \\ = 34 - 30 \\ = 4 \end{aligned}$$

At this stage it is worth mentioning the following. In mathematics we often leave out the multiplication, \times , symbol when using brackets (or when using Algebra as we will see in the next section).

So, for example, $2 \times (4 + 2)$,
can be written as $2(4 + 2)$.

USE OF VARIABLES

Letters are frequently used in mathematics, this allows us to calculate quantities using formulae over and over again.

Common letters used are x, y, t, r, v, h . Because these letters can have different values they are called *variables*.

Numbers such as 3, 45, 19 etc. are known as *constants* since they cannot have any other value.

Example:

Calculate the value of $u + at$

When $u = 2$, $a = 4$ and $t = 9$

Solution:

$$u + at = 2 + (4)(9) = 2 + 36 = 38$$

Example:

Calculate the value of $2(x + 3y) - 4xy$

When $x = 5$ and $y = 2$

Solution:

$$\begin{aligned} 2(x + 3y) - 4xy \\ = 2(5 + 3 \times 2) - 4 \times 5 \times 2 \\ = 2(5 + 6) - 20 \times 2 \\ = 2(11) - 40 \\ = 22 - 40 \\ = -18 \end{aligned}$$

Before moving on to fractions you will need to know what the reciprocal of a number is.

RECIPROCAL

The reciprocal of a number is one divided by the number.

Example:

The reciprocal of 6 is $\frac{1}{6}$

The reciprocal of 45 is $\frac{1}{45}$

That's easy!

How would the reciprocal of a fraction be written?

Let's take the fraction $\frac{1}{9}$. It's reciprocal is $1/(\frac{1}{9})$.

This is really asking how many times does $\frac{1}{9}$ divide in to 1 and of course the answer is 9!

So, the reciprocal of $\frac{1}{9}$ is 9.

A few more examples:

The reciprocal of $\frac{1}{6}$ is 6 (or $\frac{6}{1}$).

The reciprocal of 8 is $\frac{1}{8}$.

The reciprocal of $\frac{1}{20}$ is 20 (or $\frac{20}{1}$).

It can be seen from the above examples that the reciprocal of a fraction can be found by inverting the fraction (i.e. flipping it upside down).

What is the reciprocal of $\frac{3}{7}$? Easy, its $\frac{7}{3}$.

We will be mentioning reciprocals again when we see how to divide fractions and in *Sub-Module 02 Algebra*.

POSITIVE AND NEGATIVE NUMBERS (SIGNED NUMBERS)

Positive numbers are numbers that are greater than zero. *Negative numbers* are numbers less than zero. (**Figure 1-4**) *Signed whole numbers* are also called integers.

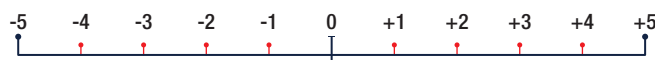


Figure 1-4. Scale of signed numbers.

ADDITION OF POSITIVE AND NEGATIVE NUMBERS

The sum (addition) of two positive numbers is positive. The sum (addition) of two negative numbers is negative. The sum of a positive and a negative number can be positive or negative, depending on the values of the numbers. A good way to visualize a negative number is to think in terms of debt. If you are in debt by \$100 (or, -100) and you add \$45 to your account, you are now only \$55 in debt (or -55). Therefore: $-100 + 45 = -55$.

Example:

The weight of an aircraft is 2 000 pounds. A radio rack weighing 3 pounds and a transceiver weighing 10 pounds are removed from the aircraft. What is the new weight? For weight and balance purposes, all weight removed from an aircraft is given a minus sign, and all weight added is given a plus sign.

$$2\,000 + -3 + -10 = 2\,000 + -13 = 1\,987$$

Therefore, the new weight is 1 987 pounds.

SUBTRACTION OF POSITIVE AND NEGATIVE NUMBERS

To subtract positive and negative numbers, first change the "-" (subtraction symbol) to a "+" (addition symbol), and change the sign of the second number to its opposite

(that is, change a positive number to a negative number or vice versa). Finally, add the two numbers together.

Example:

The daytime temperature in the city of Denver was 6° below zero (-6°). An airplane is cruising at 15 000 feet above Denver. The temperature at 15 000 feet is 20° colder than in the city of Denver. What is the temperature at 15 000 feet?

$$\text{Subtract 20 from -6: } -6 - 20 = -6 + -20 = -26$$

The temperature is -26°, or 26° below zero at 15 000 feet above the city.

MULTIPLICATION OF POSITIVE AND NEGATIVE NUMBERS

The product of two negative numbers is always positive. The product of a positive and a negative number is always negative.

Examples:

$$3 \times 6 = 18, \quad -3 \times 6 = -18, \quad -3 \times -6 = 18, \\ 3 \times -6 = -18$$

DIVISION OF POSITIVE AND NEGATIVE NUMBERS

The quotient of two positive numbers is always positive. The quotient of two negative numbers is always positive. The quotient of a positive and negative number is always negative.

Examples:

$$6 \div 3 = 2, \quad -6 \div 3 = -2, \quad -6 \div -3 = 2, \quad 6 \div -3 = -2$$

FRACTIONS

A *fraction* represents part of something, or, part of a whole thing. Anything can be divided into equal parts. We can refer to the number of these equal parts when we are expressing how much of the whole item we are talking about.

For example, the capacity of a fuel tank is 4 metric tons of fuel. If we divide the fuel tank into 4 equal parts ($4 \text{ metric tons} \div 4$), each part has a capacity of 1 metric ton of fuel.

We use fractions to express how much of the whole tank capacity to which we are referring.

If one of the equal parts of the tank is filled with fuel, the tank has 1 metric ton of fuel in it. A fraction that expresses this states the amount we are talking about compared to the total capacity of the tank.

This is written as follows:

$$\frac{1}{4} \text{ or } \frac{1}{4}$$

Therefore, when there is 1 metric ton of fuel in a 4 metric ton tank, the tank is $\frac{1}{4}$ full. A completely full tank would have all 4 parts full, or $\frac{4}{4}$.

The bottom number represents the number of equal parts into which a whole item has been divided. It is called the *denominator*.

The top number represents the specific amount of the whole item about which we are concerned. It is called the *numerator*.

The line between the numerator and the denominator is a division line. It shows that the numerator is divided by the denominator.

Some examples of fractions are:

$$\frac{17}{18} \quad \frac{2}{3} \quad \frac{5}{8} \quad \frac{1}{2} \quad \frac{13}{16} \quad \frac{8}{13}$$

The denominator of a fraction cannot be 0. This would be like saying the whole item we are talking about has been divided into 0 pieces.

As a result, 1 can be expressed as a fraction as any number over itself, or, any number divided by itself is equal to 1.

$$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{100}{100} = \frac{9.5}{9.5}$$

$$1 = \frac{17.598}{17.598} = \frac{5282}{5282} = \frac{-8}{-8}$$

There are names for various kinds of fractions. A *proper fraction* has a smaller numerator than the denominator.

$\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{16}$, and $\frac{7}{8}$ are all examples of proper fractions.

An *improper fraction* is a fraction in which the numerator is equal to or larger than the denominator.

$\frac{4}{4}$, $\frac{9}{8}$, $\frac{26}{21}$, and $\frac{3}{2}$ are all examples of improper fractions.

A whole number and a fraction together are known as a *mixed number*. Manipulating mixed numbers is discussed in greater detail in the next section of this chapter.

$1\frac{1}{2}$, $5\frac{1}{4}$, $10\frac{3}{4}$, and $8\frac{1}{8}$ are all examples of mixed numbers.

Fractions can be added, subtracted, multiplied and divided. But to do so, certain rules must be applied to arrive at the correct result.

FINDING THE LEAST COMMON DENOMINATOR (LCD)

To add or subtract fractions, the fractions must have a common denominator; that is, the denominators of the fractions to be added or subtracted must be the same. If the denominators are not the same initially, the fractions can be converted so that they have the same denominator. When adding and subtracting fractions, the *lowest*, or *least common denominator* (LCD) is often used because it often simplifies the answer. The LCD is the same as the LCM. We can therefore use either of the methods shown above to find the LCD.

Example:

To add $\frac{1}{5} + \frac{1}{10}$ the LCD can be found as follows:

List the multiples of 5 → 5, 10, 15, 20, 25, 30, etc...

List the multiples of 10 → 10, 20, 30, 40, 50, etc...

Choose the smallest multiple common to each list. 10 is a multiple common to both 5 and 10 and it is the smallest. Note that 20 and 30 are also common to each list of multiples. These could be used as common denominators as well. But the least common denominator, 10, is usually chosen to keep the numbers simple.

Example:

To add $\frac{1}{4} + \frac{1}{2} + \frac{1}{3}$ the LCD can be found as follows:

List the multiples of 4 → 4, 8, 12, 16, 20, 24, etc...

List the multiples of 2 → 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, etc...

List the multiples of 3 → 3, 6, 9, 12, 15, 18, 21, 24, etc...

Choose the smallest multiple common to each list. 12 is a multiple common to 4, 2, and 3 and it is the smallest.

Note that 24 is also common to each list of multiples. It could be used as a common denominator to solve the addition problem. But 12 is the least common denominator. It is used to keep the numbers involved small and to simplify the answer to the problem. We can multiply both the numerator and denominator of a fraction by the same number without changing its value.

Example:

$$\frac{5}{3} = \frac{5 \times 2}{3 \times 2} = \frac{5 \times 4}{3 \times 4} = \frac{5 \times 8}{3 \times 8} = \frac{5 \times 9}{3 \times 9}$$

Of course, we can also divide the numerator and denominator of a fraction by the same number without changing its value.

Example:

$$\frac{24}{36} = \frac{24 \div 3}{36 \div 3} = \frac{8}{12} = \frac{8 \div 2}{12 \div 2} = \frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

You can easily verify each of the examples using a calculator but remember calculators are not allowed in EASA examinations!

REDUCING FRACTIONS

A fraction needs to be reduced when it is not in "lowest terms." Lowest terms means that the numerator and denominator do not have any factors in common. That is, they cannot be divided by the same number (or factor). To reduce a fraction, determine what the common factor(s) are and divide these out of the numerator and denominator. For example, when both the numerator and denominator are even numbers, they can both be divided by 2.

$$\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

MIXED NUMBERS

A mixed number is a combination of a whole number and a fraction.

Each of the following are examples of mixed numbers:

$$3\frac{1}{5}, 2\frac{8}{9}, 6\frac{5}{7}$$

Mixed numbers can be written as improper fractions.

Example:

The number $3\frac{4}{5}$ represents 3 full parts and four fifths.

Three full parts is equivalent to 15 fifths.

So:

$$3\frac{4}{5} = \frac{15}{5} + \frac{4}{5} = \frac{15+4}{5} = \frac{19}{5}$$

ADDITION AND SUBTRACTION OF FRACTIONS

As stated, in order to add or subtract fractions, we use a common denominator.

Example:

Find the value of:

$$\frac{1}{5} + \frac{1}{6} + \frac{3}{10}$$

Solution:

The LCD in this case is 30 (list the multiples of 5, 6 and 10 if necessary).

Change each fraction so that its denominator is 30:

$$\frac{1}{5} = \frac{1 \times 6}{5 \times 6} = \frac{6}{30}$$

$$\frac{1}{6} = \frac{1 \times 5}{6 \times 5} = \frac{5}{30}$$

$$\frac{3}{10} = \frac{3 \times 3}{10 \times 3} = \frac{9}{30}$$

So the question becomes:

$$\frac{6}{30} + \frac{5}{30} + \frac{9}{30} = \frac{6+5+9}{30} = \frac{20}{30}$$

It's good practice to reduce the fraction as far as possible.

So our final step is:

$$\frac{20}{30} = \frac{20 \div 10}{30 \div 10} = \frac{2}{3}$$

Example:

Find the value of:

$$\frac{3}{5} + \frac{2}{7} + \frac{9}{2}$$

Solution:

The LCD of 2, 5 and 7 is 70

Writing each fraction with 70 as its denominator gives:

$$\frac{3}{5} = \frac{3 \times 14}{5 \times 14} = \frac{42}{70}$$

$$\frac{9}{2} = \frac{9 \times 35}{2 \times 35} = \frac{315}{70}$$

$$\frac{2}{7} = \frac{2 \times 10}{7 \times 10} = \frac{20}{70}$$

So the question becomes:

$$\frac{42}{70} + \frac{20}{70} + \frac{315}{70} = \frac{42+20+315}{70} = \frac{377}{70}$$

It is worth noting that 377 is a prime number and since its only factors are 1 and itself, $\frac{377}{70}$ cannot be reduced further, but it can be written as $5\frac{27}{70}$.

The following example involves both addition and subtraction.

Example:

Find the value of:

$$\frac{4}{5} + \frac{7}{3} - \frac{6}{15}$$

And reduce the answer it to its lowest form.

Solution:

The LCD is 15.

$$\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$

$$\frac{7}{3} = \frac{7 \times 5}{3 \times 5} = \frac{35}{15}$$

Now we calculate:

$$\frac{12}{15} + \frac{35}{15} - \frac{6}{15} = \frac{41}{15}$$

Note that $\frac{41}{15}$ is an improper fraction and since 15 divides in to 41 twice with 11 over, we can write:

$$\frac{41}{15} = 2\frac{11}{15}$$

MULTIPLICATION OF FRACTIONS

There is a very simple rule for multiplying fractions: multiply the numerators and multiply the denominators. An example will show how easy this process is.

Example:

Find the value of:

$$\frac{2}{5} \times \frac{3}{7}$$

Solution:

$$\frac{2}{5} \times \frac{3}{7} = \frac{2 \times 3}{5 \times 7} = \frac{6}{35}$$

Example:

Perform the following operation:

$$\frac{4}{7} \times \frac{5}{9}$$

Solution:

$$\frac{4}{7} \times \frac{5}{9} = \frac{4 \times 5}{7 \times 9} = \frac{20}{63}$$

Example:

Calculate the following.

$$\frac{2}{3} - \frac{5}{9} \times \frac{5}{6}$$

Solution:

Be very careful with this type of question. It is very tempting to calculate $\frac{2}{3} - \frac{5}{9}$ first, but you must follow the precedence rules.

We first calculate:

$$\frac{5}{9} \times \frac{5}{6} = \frac{5 \times 5}{9 \times 6} = \frac{25}{54}$$

Then we use a common denominator to compute:

$$\frac{2}{3} - \frac{25}{54}$$

The LCD is 54, therefore:

$$\frac{2}{3} - \frac{25}{54} = \frac{36}{54} - \frac{25}{54} = \frac{36-25}{54} = \frac{11}{54}$$

DIVISION OF FRACTIONS

To divide fractions, multiply by the reciprocal of the lower fraction.

Example:

Divide $\frac{7}{8}$ by $\frac{4}{3}$:

$$\left(\frac{7}{8} \div \frac{4}{3}\right) = \left(\frac{7}{8} \times \frac{3}{4}\right) = \left(\frac{7 \times 3}{8 \times 4}\right) = \frac{21}{32}$$

ADDITION OF MIXED NUMBERS

To add mixed numbers, add the whole numbers together. Then add the fractions together by finding a common denominator. The final step is to add the sum of the whole numbers to the sum of the fractions for the final result. Note the following example uses feet as the unit of measurement which can commonly result in the use of fractions. 1 foot is equal to 30.49 centimeters.

Example:

The cargo area behind the rear seat of a small airplane can handle solids that are $4\frac{3}{4}$ feet long. If the rear seats are removed, then $2\frac{1}{3}$ feet is added to the cargo area. What is the total length of the cargo area when the rear seats are removed?

$$4\frac{3}{4} + 2\frac{1}{3} = (4 + 2) + \left(\frac{3}{4} + \frac{1}{3}\right) = 6 + \left(\frac{9}{12} + \frac{4}{12}\right) \\ = 6\frac{13}{12} = 7\frac{1}{12} \leftarrow \text{feet of cargo room.}$$

SUBTRACTION OF MIXED NUMBERS

To subtract mixed numbers, find a common denominator for the fractions. Subtract the fractions from each other (*it may be necessary to borrow from the larger whole number when subtracting the fractions*). Subtract the whole numbers from each other. The final step is to combine the final whole number with the final fraction. Note the following example uses inches as the unit of measurement which can commonly result in the use of fractions. 1 inch is equal to 2.54 centimeters.

Example:

What is the length of the grip of the bolt shown in *Figure 1-5*? The overall length of the bolt is $3\frac{1}{2}$ inches, the shank length is $3\frac{1}{8}$ inches, and the threaded portion is $1\frac{5}{16}$ inches long. To find the grip, subtract the length of the threaded portion from the length of the shank.

To subtract, start with the fractions. A common denominator must be used. 16 is the LCD. So the problem becomes;

$$3\frac{2}{16} - 1\frac{5}{16} = \text{grip length in inches}$$

Borrowing will be necessary because $\frac{5}{16}$ is larger than $\frac{2}{16}$. From the whole number 3, borrow 1, which is actually $\frac{16}{16}$. After borrowing, the first mixed number is expressed as $2\frac{18}{16}$ and the equation is now as follows:

$$2\frac{18}{16} - 1\frac{5}{16} = \text{grip length in inches}$$

Again, to subtract, start with the fractions:

$$\frac{18}{16} - \frac{5}{16} = \frac{13}{16}$$

Then, subtract the whole numbers:

$$2 - 1 = 1$$

Therefore, the grip length of the bolt is $1\frac{13}{16}$ inches.

Note: The value for the overall length of the bolt was given in the example, but it was not needed to solve the problem. This type of information is sometimes referred to as a "distractor" because it distracts from the information needed to solve the problem.

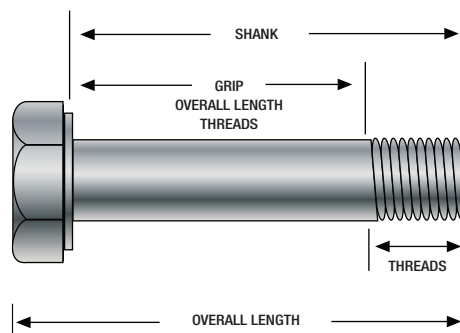


Figure 1-5. Bolt dimensions.

THE DECIMAL NUMBER SYSTEM

ORIGIN AND DEFINITION

The number system that we use every day is called the *decimal system*. The prefix in the word decimal is a Latin root for the word "ten." The decimal system probably had its origin in the fact that we have ten fingers (or digits). The decimal system has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. It is a base 10 system and has been in use for over 5 000 years. A decimal is a number with a decimal point. For example, 0.515, .10, and 462.625 are all decimal numbers. Like whole numbers, decimal numbers also have place value. The place values are based on powers of 10, as shown in *Figure 1-6*.

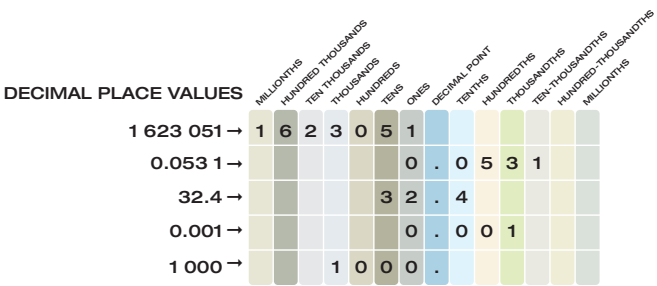


Figure 1-6. Decimal place values.

ADDITION OF DECIMAL NUMBERS

To add decimal numbers, they must first be arranged so that the decimal points are aligned vertically and according to place value. That is, adding tenths with tenths, ones with ones, hundreds with hundreds, and so forth.

Example:

Find the total resistance for the circuit diagram shown in *Figure 1-7*. As you will learn in *module 03*, total resistance of a series circuit is equal to the sum of the individual resistances. To find the total resistance, R_T , the individual resistances are added together.

$$R_T = R_1 + R_2 + R_3$$
$$R_T = 2.34 + 37.5 + .09$$

Arrange the resistance values in a vertical column so that the decimal points are aligned and then add.

$$\begin{array}{r} 2.34 \\ 37.5 \\ + .09 \\ \hline 39.93 \end{array}$$

Therefore, the total resistance, $R_T = 39.93$ ohms.

SUBTRACTION OF DECIMAL NUMBERS

To subtract decimal numbers, they must first be arranged so that the decimal points are aligned vertically and according to place value. That is, subtracting tenths from tenths, ones from ones, hundreds from hundreds, and so forth.

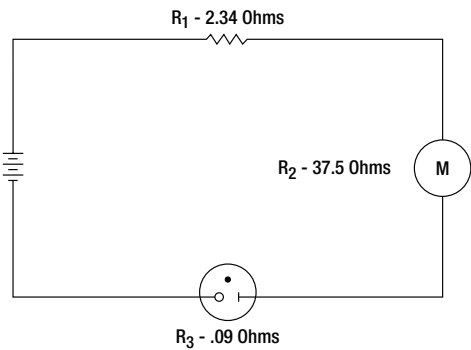


Figure 1-7. Circuit diagram.

Example:

Working again with resistance values to be studied in *Module 03*, a series circuit containing two resistors has a total resistance (R_T) of 37.272 ohms. One of the resistors (R_1) has a value of 14.88 ohms. What is the value of the other resistor (R_2)?

$$R_2 = R_T - R_1 = 37.272 - 14.88$$
$$R_2 = 37.272 - 14.88$$

Arrange the decimal numbers in a vertical column so that the decimal points are aligned and then subtract.

$$\begin{array}{r} 37.272 \\ - 14.88 \\ \hline 22.392 \end{array}$$

Therefore, the second resistor, $R_2 = 22.392$ ohms.

MULTIPLICATION OF DECIMAL NUMBERS

To multiply decimal numbers, vertical alignment of the decimal point is not required. Instead, align the numbers to the right in the same way as whole numbers

are multiplied (*with no regard to the decimal points or place values*) and then multiply. The last step is to place the decimal point in the correct place in the answer. To do this, "count" the number of decimal places in each of the numbers, add the total, and then "give" that number of decimal places to the result.

Example:

To multiply 0.2×6.03 , arrange the numbers vertically and align them to the right. Multiply the numbers, ignoring the decimal points for now.

$$\begin{array}{r} 6.03 \\ \times 0.2 \\ \hline 1.206 \end{array} \leftarrow \text{ignore the decimal points}$$

After multiplying the numbers, count the total number of decimal places in both numbers. For this example, 6.03 has 2 decimal places and 0.2 has 1 decimal place. Together there are a total of 3 decimal places. The decimal point for the answer will be placed 3 decimal places from the right. Therefore, the answer is 1.206.

Example:

Using the formula $\text{Watts} = \text{Amperes} \times \text{Voltage}$, what is the wattage of an electric drill that uses 9.45 amperes from a 120 volt source?

$$\begin{aligned} \text{Watts} &= \text{Amperes} \times \text{Voltage} \\ \text{Watts} &= 9.45 \times 120 \end{aligned}$$

Place the values to be multiplied vertically and aligned to the right.

$$\begin{array}{r} 9.45 \leftarrow 2 \text{ decimal places} \\ \times 120 \leftarrow \text{no decimal places} \\ \hline 000 \\ 1890 \\ 945 \\ \hline 1134.00 \leftarrow 2 \text{ decimal places} \end{array}$$

After multiplying the numbers, count the total number of decimal places in both numbers. For this example, 9.45 has 2 decimal places and 120 has no decimal place. Together there are 2 decimal places. The decimal point for the answer will be placed 2 decimal places from the right. Therefore, the answer is 1 134.00 watts, or 1 134 watts.

DIVISION OF DECIMAL NUMBERS

Division of decimal numbers is performed the same way as whole numbers, unless the divisor is a decimal.

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$$

When the divisor is a decimal, it must be changed to a whole number before dividing. To do this, move the decimal in the divisor to the right until there are no decimal places. At the same time, move the decimal point in the dividend to the right the same number of places. Then divide. The decimal in the quotient will be placed directly above the decimal in the dividend.

Example:

Divide 0.144 by 0.12.

$$\begin{array}{r} .12 \overline{) 0.144} = 12 \overline{) 14.4} \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Move the decimal in the divisor (0.12) two places to the right. Next move the decimal in the dividend (0.144) two places to the right. Then divide. The result is 1.2.

Example:

The wing area of an airplane is 262.6 square meters and its span is 40.4 meters. Find the mean chord of its wing using the formula: $\text{Area} \div \text{span} = \text{mean chord}$.

$$\begin{array}{r} 40.4 \overline{) 262.6} = 404 \overline{) 2626.0} \\ \underline{2424} \\ 2020 \\ \underline{2020} \\ 0.0 \end{array}$$

Move the decimal in the divisor (40.4) one place to the right. Next move the decimal in the dividend (262.6) one place to the right. Then divide. The mean chord length is 6.5 feet.

ROUNDING OFF DECIMAL NUMBERS

Occasionally, it is necessary to round off a decimal number to some value that is practical to use. For example, a measurement is calculated to be 29.4948 inches. To use this measurement, we can use the process of rounding off. A decimal is rounded off by keeping the digits for a certain number of places and discarding the rest. The degree of accuracy desired determines the number of digits to be retained. When the digit immediately to the right of the last retained digit is 5 or greater, round up by 1. When the digit immediately to the right of the last retained digit is less than 5, leave the last retained digit unchanged.

Example 1:

An actuator shaft is 4.193 8 centimeters in diameter. Round to the nearest tenth. The digit in the tenths column is a 1. The digit to the right of the 1 is a 9. Since 9 is greater than or equal to 5, "round up" the 1 to a 2. Therefore, 4.193 8 rounded to the nearest tenth is 4.2.

Example 2:

The outside diameter of a bearing is 4.193 8 centimeters. Round to the nearest hundredth. The digit in the hundredths column is a 9. The digit to the right of the 9 is a 3. Since 3 is less than 5, do not round up the 9. Therefore, 4.193 8 to the nearest hundredth is 4.19.

Example 3:

The length of a bushing is 4.193 8 centimeters. Round to the nearest thousandth. The digit in the thousandths column is a 3. The digit to the right of the 3 is an 8. Since 8 is greater than or equal to 5, "round up" the 3 to a 4. Therefore, 4.193 8 to the nearest thousandth is 4.194.

CONVERTING DECIMAL NUMBERS TO FRACTIONS

In the following discussion of converting decimal numbers to fractions, examples use inches for the units of denomination (rather than centimeters). Technicians in countries using Imperial System denomination must often work with fractions. To change a decimal number to a fraction, "read" the decimal, and then write it into a fraction just as it is read as shown below.

Example:

One oversized rivet has a diameter of 0.52 inches. Convert 0.52 to a fraction. The decimal 0.52 is read as "fifty-two hundredths."

Therefore:

$$0.52 = \frac{52}{100} \leftarrow \begin{array}{l} \text{"fifty-two"} \\ \text{"hundredths"} \end{array} \text{ which can be reduced to } \frac{13}{25}$$

A dimension often appears in a maintenance manual or on a blueprint as a decimal instead of a fraction. In order to use the dimension, it may need to be converted to a fraction. An aviation mechanic frequently uses a steel rule that is calibrated in units of $\frac{1}{64}$ of an inch. To change a decimal to the nearest equivalent common fraction, multiply the decimal by 64. The product of the decimal and 64 will be the numerator of the fraction and 64 will be the denominator. Reduce the fraction, if needed.

Example 1:

The width of a hex head bolt is 0.312 3 inches. Convert the decimal 0.312 3 to a common fraction to decide which socket would be the best fit for the bolt head. First, multiply the 0.312 3 decimal by 64:

$$0.312\ 3 \times 64 = 19.987\ 2$$

Next, round the product to the nearest whole number: $19.987\ 22 \approx 20$.

Use this whole number (20) as the numerator and 64 as the denominator: $\frac{20}{64}$.

Now, reduce $\frac{20}{64}$ to $\frac{5}{16}$.

Therefore, the correct socket would be the $\frac{5}{16}$ inch socket ($\frac{20}{64}$ reduced).

Example 2:

When accurate holes of uniform diameter are required for aircraft structures, they are first drilled approximately $\frac{1}{64}$ inch undersized and then reamed to the final desired diameter. What size drill bit should be selected for the undersized hole if the final hole is reamed to a diameter of 0.763 inches? First, multiply the 0.763 decimal by 64.

$$0.763 \times 64 = 48.832$$

Next, round the product to the nearest whole number: $48.832 \approx 49$.

Use this number (49) as the numerator and 64 as the denominator: $\frac{49}{64}$ is the closest fraction to the final remaining diameter of 0.763 inches. To determine the drill size for the initial undersized hole, subtract $\frac{1}{64}$ inch from the finished hole size.

$$\frac{49}{64} - \frac{1}{64} = \frac{48}{64} = \frac{3}{4}$$

Therefore, a $\frac{3}{4}$ inch drill bit should be used for the initial undersized holes.

Calculator tip: Numerator (top number) \div Denominator (bottom number) = the decimal equivalent of the fraction. Some fractions when converted to decimals produce a repeating decimal.

Example:

$$\frac{1}{3} = 1 \div 3 = 3 \overline{)1.00} = \overline{.3} \text{ or } .33$$

$\begin{array}{r} .33 \\ 3 \overline{)1.00} \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 1 \end{array}$

This decimal can be represented with bar, or can be rounded. (A bar indicates that the number(s) beneath it are repeated to infinity.)

Examples of repeating decimals:

$$.212121\ldots = .21$$

$$.6666\ldots = .7 \text{ or } .67$$

$$.254254\ldots = .254$$

DECIMAL EQUIVALENT CHART

Figure 1-8 is a fraction to decimal to millimeter conversion chart. Measurements starting at $\frac{1}{64}$ inch up to 1 inch have been converted to decimal numbers and to millimeters.

fraction	decimal	mm	fraction	decimal	mm
1/64	0.015	0.396	33/64	0.515	13.096
1/32	0.031	0.793	17/32	0.531	13.493
3/64	0.046	1.190	35/64	0.546	13.890
1/16	0.062	1.587	39/64	0.562	14.287
5/64	0.078	1.984	37/64	0.578	14.684
3/32	0.093	2.381	19/32	0.593	15.081
7/64	0.109	2.778	39/64	0.609	15.478
1/8	0.125	3.175	5/8	0.625	15.875
9/64	0.140	3.571	41/64	0.640	16.271
5/32	0.156	3.968	21/32	0.656	16.668
11/64	0.171	4.365	43/64	0.671	17.065
5/16	0.187	4.762	11/16	0.687	17.462
13/64	0.203	5.159	45/64	0.703	17.859
7/32	0.218	5.556	23/32	0.718	18.256
15/64	0.234	5.953	47/64	0.734	18.653
1/4	0.25	6.35	3/4	0.75	19.05
17/64	0.265	6.746	49/64	0.765	19.446
9/32	0.281	7.143	25/32	0.781	19.843
19/64	0.296	7.540	51/64	0.796	20.240
5/16	0.312	7.937	13/16	0.812	20.637
21/64	0.328	8.334	53/64	0.828	21.034
11/32	0.343	8.731	27/32	0.843	21.431
23/64	0.359	9.128	55/64	0.859	21.828
3/8	0.375	9.525	7/8	0.875	22.225
25/64	0.390	9.921	57/64	0.890	22.621
13/32	0.406	10.318	29/32	0.906	23.018
27/64	0.421	10.715	59/64	0.921	23.415
7/16	0.437	11.112	15/16	0.937	23.812
29/64	0.453	11.509	61/64	0.953	24.209
15/32	0.468	11.906	31/32	0.968	24.606
31/64	0.484	12.303	63/64	0.984	25.003
1/2	0.5	12.7	1	1	25.4

Figure 1-8. A fractions to decimals to millimeters conversion chart.

RATIO

A *ratio* is the result of one number or quantity divided by another. Ratios are a simple mathematical tool that allow meaningful comparisons.

A ratio may be expressed in three ways: as a fraction, with a colon, or with the word "to." For example, a gear ratio of 5:7 can be expressed as any of the following:

$\frac{5}{7}$ or 5:7 or 5 to 7

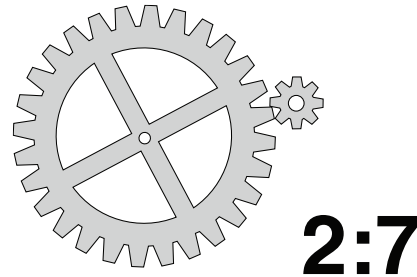


Figure 1-9. Gear ratio.

AVIATION APPLICATIONS OF RATIOS

Ratios have widespread application in the field of aviation.

Compression ratio on a reciprocating engine is the ratio of the volume of a cylinder with the piston at the bottom of its stroke to the volume of the cylinder with the piston at the top of its stroke. For example, a typical compression ratio might be 10:1 (or 10 to 1).

Aspect ratio is the ratio of the length (or span) of an airfoil to its width (or chord). A typical aspect ratio for a commercial airliner might be 7:1 (or 7 to 1).

Air-fuel ratio is the ratio of the weight of the air to the weight of fuel in the mixture being fed into the cylinders of a reciprocating engine. For example, a typical air-fuel ratio might be 14.3:1 (or 14.3 to 1).

Glide ratio is the ratio of the forward distance traveled to the vertical distance descended when an aircraft is operating without power. For example, if an aircraft descends 300 meters while it travels through the air for a distance of 3 kilometers, it has a glide ratio of 3 000:300 which can be reduced to 10:1 (or 10 to 1).

Gear ratio is the number of teeth each gear represents when two gears are used in an aircraft component. In **Figure 1-9**, the pinion gear has 8 teeth and the spur gear has 28 teeth. The gear ration is 8:28 or 2:7.

When two gears are used in an aircraft component, the rotational speed of each gear is represented as a *speed ratio*. As the number of teeth in a gear decreases, the rotational speed of that gear increases, and vice-versa. Therefore, the speed ratio of two gears is the inverse (or opposite) of the gear ratio. If two gears have a gear ratio of 2:9, then their speed ratio is 9:2.

Example:

A pinion gear with 10 teeth is driving a spur gear with 40 teeth. The spur gear is rotating at 160 revolutions per minute (rpm). Determine the speed of the pinion gear.

$$\frac{\text{Teeth in Pinion Gear}}{\text{Teeth in Spur Gear}} = \frac{\text{Speed of Spur Gear}}{\text{Speed of Pinion Gear}}$$

$$\frac{10 \text{ teeth}}{40 \text{ teeth}} = \frac{160 \text{ rpm}}{S_p \text{ (speed of pinion gear)}}$$

To solve for S_p , multiply 40×160 , then divide by 10. The speed of the pinion gear is 640 rpm.

Example:

If the cruising speed of an airplane is 400 km/h and its maximum speed is 500 km/h, what is the ratio of cruising speed to maximum speed? First, express the cruising speed as the numerator of a fraction whose denominator is the maximum speed. Next, reduce the resulting fraction to its lowest terms.

$$\text{Ratio} = \frac{400}{500}$$

Therefore, the ratio of cruising speed to maximum speed is 4:5.

$$\text{Ratio} = \frac{400}{500} = \frac{4}{5}$$

Another common use of ratios is to convert any given ratio to an equivalent ratio with a denominator of 1.

Example:

Express the ratio 9:5 as a ratio with a denominator of 1.

$$R = \frac{9}{5} = \frac{?}{1} \quad \text{Since } 9 \div 5 = 1.8, \text{ then } \frac{9}{5} = \frac{1.8}{1}$$

Therefore, 9:5 is the same ratio as 1.8:1. In other words, 9 to 5 is the same ratio as 1.8 to 1.

PROPORTION

A *proportion* is a statement of equality between two or more ratios.

Example:

$$\frac{3}{4} = \frac{6}{8} \text{ or } 3:4 = 6:8$$

This proportion is read as, "3 is to 4 as 6 is to 8."

SOLVING PROPORTIONS

Normally when solving a proportion, three quantities will be known, and the fourth will be unknown. To solve for the unknown, multiply the two numbers along the diagonal and then divide by the third number.

Example 1:

Solve for x in the proportion given below.

$$\frac{65}{80} = \frac{x}{100}$$

Solution:

Multiply both sides by 100

$$100 \times \frac{65}{80} = \frac{x}{100} \times 100$$

Therefore,

$$\frac{6500}{80} = x$$

Finally,

$$81.25 = x$$

Example 2:

An airplane flying a distance of 300 kilometers used 20 liters of gasoline. How many liters will it need to travel 600 kilometers?

The ratio here is: "kilometers to liters;" therefore, the proportion is set up as:

$$\frac{\text{Kilometers}}{\text{Liters}} = \frac{300}{20} = \frac{600}{L}$$

$$\text{Solve for } L: (600 \times 20) \div 300 = L$$

$$12\,000 \div 300 = 40$$

Therefore, to fly 600 kilometers, 40 liters of fuel will be required.

AVERAGE VALUE

An *average* is a single value that typifies a list of values. If all the numbers in the list are the same, then this number is the *average*.

To find the average of a list of values, add together all of the values in the list and divide the sum by the number of values in the list.

Example:

To find the average of the following list of numbers, add the numbers together and divide the sum by 4.

12, 10, 8, 14

$$12 + 10 + 8 + 14 = 44, \frac{44}{4} = 11$$

Therefore the average value of the numbers in the list is 11.

$$\text{The average of 2 and 8 is } \frac{2+8}{2} = 5$$

Switching the order of 2 and 8 to read 8 and 2 does not change the resulting value obtained for average. In the next *Sub-Module, Algebra* will be discussed. The algebraic expression for determining average (or mean) is:

$$\text{Average} = \frac{a_1 + a_2 + \dots + a_n}{n}$$

Where n numbers are given, each number is denoted by a_i , and where $i = 1, 2, \dots, n$.

PERCENTAGE

Percentage means "parts out of one hundred." The percentage sign is "%". Ninety percent is expressed as 90% (= 90 parts out of 100). The decimal 0.90 equals $\frac{90}{100}$, or 90 out of 100 or 90%.

EXPRESSING A DECIMAL NUMBER AS A PERCENTAGE

To express a decimal number in percent, move the decimal point two places to the right (*adding zeros if necessary*) and then affix the percent symbol.

Example:

Express the following decimal numbers as a percent.

$$.90 = \frac{90}{100} = 90\%$$

$$.5 = \frac{50}{100} = 50\%$$

$$1.25 = \frac{125}{100} = 125\%$$

$$.335 = \frac{33.5}{100} = 33.5\%$$

EXPRESSING A PERCENTAGE AS A DECIMAL NUMBER

Sometimes it may be necessary to express a percentage as a decimal number. To express a percentage as a decimal number, move the decimal point two places to the left and drop the % symbol.

Example:

Express the following percentages as decimal numbers.

$$90\% = \frac{90}{100} = 0.90$$

$$50\% = \frac{50}{100} = 0.50$$

$$5\% = \frac{5}{100} = 0.05$$

$$150\% = \frac{150}{100} = 1.50$$

EXPRESSING A FRACTION AS A PERCENTAGE

To express a fraction as a percentage as shown earlier, multiply the fraction by 100, change the fraction to a decimal number (by dividing the numerator by the denominator), and then write the % sign.

Example:

Express the fraction $\frac{5}{8}$ as a percentage.

Solution:

$$\frac{5}{8} \times 100 = \frac{500}{8} = 62.5\%$$

FINDING A PERCENTAGE OF A GIVEN NUMBER

This is the most common type of percentage calculation and thankfully the simplest to work out. Just remember that percent means per hundred (i.e. out of 100) and when we say 'of' in mathematics we multiply. (For example one half of 6 is $\frac{1}{2} \times 6 = 3$!)

Example:

In a shipment of 80 wingtip lights, 15% of the lights were defective. How many of the lights were defective?

Solution:

We want to find 15% of 80.

Write the question as:

$$\frac{15}{100} \times 80 = \frac{1200}{100} = 12$$

FINDING WHAT PERCENTAGE ONE NUMBER IS OF ANOTHER

Say you get 22 marks out of a total of 40 in a Mathematics test. How would you find your mark as a percentage?

You write it as a fraction i.e. $\frac{22}{40}$ and multiply by 100.

Your grade as a % is then:

$$\frac{22}{40} \times 100 = 55\%$$

Example:

A small engine rated at 12 horsepower is found to be delivering only 10.75 horsepower.

What is the motor efficiency expressed as a percent?

Solution:

The engine is delivering 10.75 out of a possible 12.

Write this as a fraction i.e. $\frac{10.75}{12}$ and multiply by 100.

$$\frac{10.75}{12} \times 100 = \frac{1075}{12} = 89.58$$

Therefore, the motor efficiency is 89.58%.

FINDING A NUMBER WHEN A PERCENTAGE OF IT IS KNOWN

Example:

Eighty ohms represents 52% of a microphone's total resistance. Find the total resistance of this microphone.

Solution:

It is a standard practice in Mathematics to let x represent an unknown quantity.

We know that 52% of x is 80, so we write this as:

$$\frac{52}{100} \times x = 80$$

Multiple both sides of this equation by 100 and divide both sides by 52 to get:

$$x = 80 \times \frac{100}{52}$$

Finally:

$$x = \frac{8\,000}{52} = 153.846 \text{ ohms}$$

POWERS AND INDICES

The *power* (or *exponent*) of a number is a shorthand method of indicating how many times a number, called the *base*, is multiplied by itself. Powers and exponents are also known as *indices*. For example, 3^4 means "3 to the power of 4." That is, 3 multiplied by itself 4 times. The 3 is the base and 4 is the power.

Example:

$$2^3 = 2 \times 2 \times 2 = 8$$

Read "two to the third power equals 8."

$$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100\,000$$

Read "ten to the fifth power equals 100 000."

When a number has a power of 3, it is commonly referred to as cubed. For example, 7^3 is read as "seven cubed" or "seven to the third power." To remember this, think about how a cube has three dimensions: length, width, and depth.

Any non-zero number raised to the zero power always equals 1.

Example:

$$7^0 = 1, \quad 181^0 = 1, \quad (-24)^0 = 1$$

NEGATIVE POWERS

A number with a negative power equals its reciprocal with the same power made positive.

Example:

The number 2^{-3} is read as "2 to the negative 3rd power," and is calculated by:

$$2^{-3} = \frac{1}{2^3} = \frac{1}{2 \times 2 \times 2} = \frac{1}{8}$$

SQUARES AND CUBES

When a number has a power of 2, it is commonly referred to as *squared*. For example, 7^2 is read as "seven squared" or "seven to the second power." To remember this, think about how a square has two dimensions: length and width.

When using a calculator to raise a negative number to a power, always place parentheses around the negative number (*before raising it to a power*) so that the entire number gets raised to the power.

LAW OF EXPONENTS

When multiplying numbers with *powers*, the powers can be added as long as the bases are the same.

Example:

$$\begin{aligned} 3^2 \times 3^4 &= (3 \times 3) \times (3 \times 3 \times 3 \times 3) = \\ &3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6 \\ \text{or, } 3^2 \times 3^4 &= 3^{(2+4)} = 3^6 \end{aligned}$$

When dividing numbers with powers, the powers can be subtracted as long as the bases are the same.

Example:

$$\begin{aligned} 10^4 \div 10^2 &= \frac{10 \times 10 \times 10 \times 10}{10 \times 10} = \frac{\cancel{10} \times \cancel{10} \times 10 \times 10}{\cancel{10} \times \cancel{10}} = 10 \times 10 = 10^2 \\ \text{or } 10^4 \div 10^2 &= 10^{(4-2)} = 10^2 \end{aligned}$$

POWERS OF TEN

Because we use the decimal system of numbers, *powers of ten* are frequently seen in everyday applications. For example, *scientific notation* uses powers of ten. Also, many aircraft drawings are scaled to powers of ten. **Figure 1-10** gives more information on the powers of ten and their values.

POWERS OF TEN	EXPANSION	VALUE
POSITIVE EXPONENTS		
10^6	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	1 000 000
10^5	$10 \times 10 \times 10 \times 10 \times 10$	100 000
10^4	$10 \times 10 \times 10 \times 10$	10 000
10^3	$10 \times 10 \times 10$	1 000
10^2	10×10	100
10^1	10	10
10^0		1
NEGATIVE EXPONENTS		
10^{-1}	1/10	1/10 = 0.1
10^{-2}	1/(10 × 10)	1/100 = 0.01
10^{-3}	1/(10 × 10 × 10)	1/1 000 = 0.001
10^{-4}	1/(10 × 10 × 10 × 10)	1/10 000 = 0.0001
10^{-5}	1/(10 × 10 × 10 × 10 × 10)	1/100 000 = 0.00001
10^{-6}	1/(10 × 10 × 10 × 10 × 10 × 10)	1/1 000 000 = 0.000001

Figure 1-10. Powers of ten.

ROOTS

A *root* is a number that when multiplied by itself a specified number of times will produce a given number.

The two most common roots are the *square root* and the *cube root*. For more examples of roots, see the chart in **Figure 1-11**.

SQUARE ROOTS

The square root of 25, written as $\sqrt{25}$, equals 5. That is, when the number 5 is squared (multiplied by itself), it produces the number 25. The symbol $\sqrt{\quad}$ is called a *radical sign*. Finding the square root of a number is the most common application of roots. Numbers whose square roots are whole numbers are called *perfect squares*. The first ten perfect squares are: 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100. The square root of each of these numbers is 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10, respectively.

Examples:

$$\sqrt{36} = 6 \text{ and } \sqrt{81} = 9$$

To find the square root of a number that is not a perfect square, use either a calculator or the estimation method. A longhand method does exist for finding square roots, but with the advent of calculators and because of its lengthy explanation, it is not included in this module.

The estimation method uses the knowledge of perfect squares to approximate the square root of a number.

Example:

Find the square root of 31. Since 31 falls between the two perfect roots 25 and 36, we know that $\sqrt{31}$ must be between $\sqrt{25}$ and $\sqrt{36}$. Therefore, $\sqrt{31}$ must be greater than 5 and less than 6 because $\sqrt{25} = 5$ and $\sqrt{36} = 6$. If you estimate the square root of 31 at 5.5, you are close to the correct answer. The square root of 31 is actually 5.568.

NUMBER	SQUARE	CUBE	SQUARE ROOT	CUBE ROOT	CIRCUMFERENCE	AREA
Number (N)	N Squared (N^2)	N Cubed (N^3)	Square Root of N (\sqrt{N})	Cube Root of N ($\sqrt[3]{N}$)	Circumference of a Circle with Diameter = N	Area of a Circle with Diameter = N
1	1	1	1.000	1.000	3.142	0.785
2	4	8	1.414	1.260	6.283	3.142
3	9	27	1.732	1.442	9.425	7.069
4	16	64	2.000	1.587	12.566	12.566
5	25	125	2.236	1.710	15.708	19.635
6	36	216	2.449	1.817	18.850	28.274
7	49	343	2.646	1.913	21.991	38.484
8	64	512	2.828	2.000	25.133	50.265
9	81	729	3.000	2.080	28.274	63.617
10	100	1,000	3.162	2.154	31.416	78.540
11	121	1,331	3.317	2.224	34.558	95.033
12	144	1,728	3.464	2.289	37.699	113.01
13	169	2,197	3.606	2.351	40.841	132.73
14	196	2,744	3.742	2.410	43.982	153.94
15	225	3,375	3.873	2.466	47.124	176.71
16	256	4,096	4.000	2.520	50.265	201.06
17	289	4,913	4.123	2.571	53.407	226.98
18	324	5,832	4.243	2.621	56.549	254.47
19	361	6,859	4.359	2.668	59.690	283.53
20	400	8,000	4.472	2.714	62.832	314.16
21	441	9,261	4.583	2.759	65.973	346.36
22	484	10,648	4.690	2.802	69.115	380.13
23	529	12,167	4.796	2.844	72.257	415.48
24	576	13,824	4.899	2.885	75.398	452.39
25	625	15,625	5.000	2.924	78.540	490.87
26	676	17,576	5.099	2.963	81.681	530.93
27	729	19,683	5.196	3.000	84.823	572.55
28	784	21,952	5.292	3.037	87.965	615.75
29	841	24,389	5.385	3.072	91.106	660.52
30	900	27,000	5.477	3.107	94.248	706.86
31	961	29,791	5.568	3.141	97.389	754.77
32	1,024	32,768	5.657	3.175	100.531	804.25
33	1,089	35,937	5.745	3.208	103.672	855.30
34	1,156	39,304	5.831	3.240	106.814	907.92
35	1,225	42,875	5.916	3.271	109.956	962.11
36	1,296	46,656	6.000	3.302	113.097	1017.88
37	1,369	50,653	6.083	3.332	116.239	1075.21
38	1,444	54,872	6.164	3.362	119.380	1134.11
39	1,521	59,319	6.245	3.391	122.522	1194.59
40	1,600	64,000	6.325	3.420	125.664	1256.64
41	1,681	68,921	6.403	3.448	128.805	1320.25
42	1,764	74,088	6.481	3.476	131.947	1385.44
43	1,849	79,507	6.557	3.503	135.088	1452.20
44	1,936	85,184	6.633	3.530	138.230	1520.53
45	2,025	91,125	6.708	3.557	141.372	1590.43
46	2,116	97,336	6.782	3.583	144.513	1661.90
47	2,209	103,823	6.856	3.609	147.655	1734.94
48	2,304	110,592	6.928	3.634	150.796	1809.56
49	2,401	117,649	7.000	3.659	153.938	1885.74
50	2,500	125,000	7.071	3.684	157.080	1963.49

Figure 1-11. Functions of numbers.

CUBE ROOTS

The cube root of 125, written as $\sqrt[3]{125}$, equals 5. That is, when the number 5 is cubed ($5 \times 5 \times 5$), it produces the number 125. It is common to confuse the "cube" of a number with the "cube root" of a number. For clarification, the cube of $27 = 27^3 = 27 \times 27 \times 27 = 19\,683$. However, the cube root of $27 = \sqrt[3]{27} = 3$.

FRACTIONAL INDICIES

Another way to write a root is to use a fraction as the index (or exponent) instead of the radical sign. The square root of a number is written with a $\frac{1}{2}$ as the exponent instead of a radical sign. The cube root of a number is written with an exponent of $\frac{1}{3}$ and the fourth root with an exponent of $\frac{1}{4}$ and so on.

Examples:

$$\sqrt{31} = 31^{1/2} \quad \sqrt[3]{125} = 125^{1/3} \quad \sqrt[4]{16} = 16^{1/4}$$

SCIENTIFIC AND ENGINEERING NOTATION

Scientific notation is used as a type of shorthand to express very large or very small numbers. It is a way to write numbers so that they do not take up as much space on the page. The format of a number written in scientific notation has two parts. The first part is a number greater than or equal to 1 and less than 10 (for example, 2.35). The second part is a power of 10 (for example, 10^6). The number 2 350 000 is expressed in scientific notation as 2.35×10^6 . It is important that the decimal point is always placed to the right of the first digit.

Notice that very large numbers always have a positive power of 10 and very small numbers always have a negative power of 10.

Example:

The velocity of the speed of light is over 186 000 000 mph. This can be expressed as 1.86×10^8 mph in scientific notation. The mass of an electron is approximately:

0.000 000 000 000 000 000 000 000 911 grams.

This can be expressed in scientific notation as:

9.11×10^{-28} grams.

Engineering notation is a version of scientific notation in which the exponent of ten must be divisible by three (i.e., they are powers of a thousand, but written as, for example, 10^6 instead of $100\,0^2$).

CONVERTING NUMBERS FROM STANDARD NOTATION TO SCIENTIFIC OR ENGINEERING NOTATION

Note that when any number is written without a decimal point, it is implied that the decimal point is to the right of the last digit.

To change the format of a number from standard notation to scientific notation, move the decimal point to the position between the first and the second digits. Count the number of places the decimal point is moved. Including the new position of the decimal point, write the number without the string of zero place holders. Then write $\times 10$ and raise 10 to the power equal to the number of places the decimal point was moved. If the decimal is moved to the left such as when working with a large number, the power is positive. If the decimal point is moved to the right such as when working with a small number, the power is negative.

Example 1:

Convert 1 244 000 000 000 to scientific notation.

The decimal point, although not written, is located to the right of the last zero. It must be moved to the left 12 places to be positioned between the first and the second digit. Writing the number without the string of zeros, we now have 1.244. Then write $\times 10^{12}$ because the decimal point was moved 12 places to the left. Thus, 1 244 000 000 in standard notation becomes 1.244×10^{12} in scientific notation.

Example 2:

Convert 0.000 000 457 from standard notation to scientific notation. The decimal point must be moved to the right 7 places to be between the first and second digits that are not zeros. Writing the number without the string of zeros, we now have 4.57. Then write $\times 10^{-7}$ because the decimal point was moved 7 places to the right.

Thus, 0.000 000 457 in standard notation becomes 4.57×10^{-7} in scientific notation.

Example 3:

Write the numbers 26 100, using engineering format. Move the point three places to the left, so that $26\ 100 = 26.1 \times 10^3$

CONVERTING NUMBERS FROM SCIENTIFIC OR ENGINEERING NOTATION TO STANDARD NOTATION

To convert numbers from scientific notation to standard notation, the decimal point must be moved the number of places indicated by the power of 10. For large numbers, the power of 10 is positive and the decimal must be moved to the right. For small numbers, the power of 10 is negative and the decimal must be moved to the left. When moving the decimal point, zeros must be added to preserve place value.

Example:

Convert 3.68×10^7 from scientific notation to standard notation, as follows. To convert from scientific notation to standard notation, move the decimal place 7 places to the right. $3.68 \times 10^7 = 36\ 800\ 000$. Another way to think about the conversion is $3.68 \times 10^7 = 3.68 \times 10\ 000\ 000 = 36\ 800\ 000$.

Example:

Convert $7.154\ 3 \times 10^{-10}$ from scientific notation to standard notation. Move the decimal place 10 places to the left: $7.154\ 3 \times 10^{-10} = .000\ 000\ 000\ 715\ 43$. Another way to think about the conversion is $7.154\ 3 \times 10^{-10} = 7.154\ 3 \times .000\ 000\ 000\ 1 = .000\ 000\ 000\ 715\ 43$

When converting, remember that large numbers always have positive powers of ten and small numbers always have negative powers of ten. Refer to **Figure 1-12** to determine which direction to move the decimal point.

ADDITION, SUBTRACTION, MULTIPLICATION, DIVISION OF SCIENTIFIC AND ENGINEERING NUMBERS

To add, subtract, multiply, or divide numbers in scientific notation, change the scientific notation number back to standard notation. Then add, subtract, multiply or divide the standard notation numbers. After the computation, change the final standard notation number back to scientific notation.

CONVERSION	LARGE NUMBERS WITH POSITIVE POWERS OF 10	SMALL NUMBERS WITH NEGATIVE POWERS OF 10
From standard notation to scientific notation	Move decimal place to the left	Move decimal place to the right
From scientific notation to standard notation	Move decimal place to the right	Move decimal place to the left

Figure 1-12. Converting between scientific and standard notation.

DENOMINATED NUMBERS

ADDITION OF DENOMINATED NUMBERS

Numbers that have a unit of measure associated with them, such as meter, kilowatt, kilogram, liter, etc., are denominated numbers. A list of some common base units and symbols is given in **Figure 1-13**.

Base Units and Symbols		
Symbol for Unit	Base Unit	Base Quantity
m	meter	length (l)
kg	kilogram	mass (m)
s	second	time (t)
A	Ampere	electric current (I)
K	Kelvin	temperature

Figure 1-13. Base units and symbols table.

The word *denominate* means to give a name to the numbers; they are not just abstract symbols. Non-SI units of denomination are used in some examples in this section as they exemplify the intricacies inherent in performing arithmetic operations with denominated numbers.

Example:
Add 6 feet, 8 inches to 4 feet, 5 inches:

$$\begin{array}{r} 6 \text{ ft. } 8 \text{ in.} \\ + 4 \text{ ft. } 5 \text{ in.} \\ \hline 10 \text{ ft. } 13 \text{ in.} \end{array}$$

Since 13 in. is the equivalent of 1 ft. 1 in. regroup the answer as 11 ft. 1 in.

Example:
Add 20 degrees 44 minutes 6 seconds to 13 degrees 22 minutes 5 seconds (20°44'6" and 13°22'5").

$$\begin{array}{r} 20^{\circ}44'6'' \\ + 13^{\circ}22'5'' \\ \hline 33^{\circ}66'11'' \end{array}$$

This answer is regrouped as: 34 degrees, 6 minutes, 11 seconds. (34°6'11") since 66 minutes = 1° 6'.

Denominated numbers must be expressed in units of the same kind in order to be manipulated and combined. For instance, the sum of 6 kilowatts plus 1 watt is not 7 kilowatts nor is it 7 watts. The addition can only be performed by converting one of the numbers so it is denominated the same as the other. 6 kilowatts = 6 000 watts. With this conversion, the numbers are denominated the same and addition can proceed.

Note that when addition is performed with denominated numbers, the denomination is carried throughout the entire addition process to the sum without changes. However, the sum can be converted to the most relevant denomination.

$$6\,000 \text{ watts} + 7 \text{ watts} = 6\,007 \text{ watts}$$

This can also be expressed as: 6.007 kilowatts

SUBTRACTION OF DENOMINATED NUMBERS

Subtraction of denominated numbers follows the same rules as addition of denominated numbers. Borrowing or reconfiguring the values may be required but must not affect the overall value of the quantities presented.

Example:
Subtract 16 deg. 8 min. 2 sec. from 28 deg. 4 min. 3 sec. (16°8'2" and 28°4'3").

$$\begin{array}{r} 28^{\circ}4'3'' \\ - 16^{\circ}8'2'' \\ \hline \end{array}$$

In order to subtract 8 min from 4 min we regroup as follows:

Note that the denomination does not change from the problem to the quotient.

$$\begin{array}{r} 27^{\circ}64'3'' \\ - 16^{\circ}8'2'' \\ \hline 11^{\circ}56'1'' \end{array}$$

Usually when denominated numbers are written using Engineering Format a symbol or prefix is used instead of the power of 10. The common prefixes and symbols are shown in **Figure 1-14**.

Example:
Write 517.3 nanograms, first using the appropriate symbol, then using engineering format.

Solution:

$$517.3 \text{ nanograms} = 517.3 \text{ ng} = 517.3 \times 10^{-9} \text{ g.}$$

Common Prefixes And Symbols		
10 ¹²	T	tera
10 ⁹	G	giga
10 ⁶	M	mega
10 ³	k	kilo
10 ⁻³	m	milli
10 ⁻⁶	μ	micro
10 ⁻⁹	n	nano
10 ⁻¹²	p	pico

Figure 1-14. Common prefixes and symbols table.

Example:

Find the sum of the following, giving your answer in the basic unit. $i = 3.5A + 0.42kA + 81.3 \text{ mA}$

Solution:

$$\begin{aligned} i &= 3.5A + (0.42 \times 10^3)A + (81.3 \times 10^{-3})A \\ &= (3.5 + 420 + 0.0813)A \\ &= 423.5813A \end{aligned}$$

Example:

Find the sum of the following, giving your answer in the basic unit. $f = 3500 \text{ Hz} + 6 \text{ GHz} + 39 \text{ MHz}$

Solution:

$$\begin{aligned} f &= 3500 \text{ Hz} + 6 \times 10^9 \text{ Hz} + 39 \times 10^6 \text{ Hz} \\ &= (3500 + 6000000000 + 39000000) \text{ Hz} \\ &= 6039003500 \text{ Hz} \end{aligned}$$

Example:

Convert 3456MHz to the appropriate kilo unit.

Solution:

$$\begin{aligned} 3456 \text{ MHz} &= 3456 \times 10^6 \text{ Hz} \\ &= 3456 \times 10^3 \times 10^3 \text{ Hz} \\ &= 3456 \times 10^3 \text{ kHz} \\ &= 3456000 \text{ kHz} \end{aligned}$$

MULTIPLICATION OF DENOMINATED NUMBERS

When multiplying denominated numbers by non-denominated numbers, multiply differently denominated numbers separately, then, examine the products for any combining that might simplify the expression of the values.

Example:

$3 \times (5 \text{ yards}, 2 \text{ feet}, 6 \text{ inches})$

$$\begin{array}{r} 5 \text{ yd. } 2 \text{ ft. } 6 \text{ in.} \\ \times \quad \quad \quad 3 \\ \hline 15 \text{ yd. } 6 \text{ ft. } 18 \text{ in.} \end{array} = 17 \text{ yd. } 1 \text{ ft. } 6 \text{ in.}$$

Simplifying:

18 inches = 1 foot 6 inches. Thus, the product can be re-written by combining feet: 15 yards, 7 feet, 6 inches. However, simplifying further, 7 feet = 2 yards + 1 foot.

Combining numbers denominated in feet and yards into yards, the product can now be written 17 yards, 1 foot, 6 inches.

When one denominated number is multiplied by another, a question arises concerning the products of the units of measurement. The product of one unit times another of the same kind is one square unit. For example, 1 meter \times 1 meter = 1 square meter (*abbreviated 1 m²*). If it becomes necessary to multiply such numbers as 2 yards, 2 feet \times 6 yards, 2 feet, the foot units may be converted to fractions of a yard or yards may be converted to feet so that the entire multiplication problem takes place using a single denomination.

Example: Multiply $4 \text{ yd. } 2 \text{ ft.} \times 3 \text{ yd. } 2 \text{ ft.}$

The foot units may be converted to fractions of a yard, as follows:

$$\begin{aligned} (4 \text{ yd. } 2 \text{ ft.}) \times (3 \text{ yd. } 2 \text{ ft.}) &= 4\frac{2}{3} \text{ yd.} \times 3\frac{2}{3} \text{ yd.} = \\ \frac{154}{9} &= \text{sq. yd.} = 17\frac{1}{9} \text{ sq. yd.} \end{aligned}$$

Alternately, the values presented in yards could be converted to feet before the arithmetic is performed for an answer that results in square feet:

$$\begin{aligned} (4 \text{ yd. } 2 \text{ ft.}) \times (3 \text{ yd. } 2 \text{ ft.}) &= 14 \text{ ft.} \times 11 \text{ ft.} \\ &= 154 \text{ sq. ft.} \end{aligned}$$

DIVISION OF DENOMINATED NUMBERS

Division of denominated numbers is accomplished easily by converting all denominated numbers into the same units of denomination and performing the arithmetic. If division is by a non-denominated number, the denomination of the quotient will be the same as the units into which the divisor and dividend were converted before the division took place.

Example:

$$\begin{aligned} 4 \text{ liters, } 400 \text{ milliliters} / 2 &= 4 \text{ liters} + .4 \text{ liters} / 2 \\ &= 4.4 \text{ liters} / 2 = 2.2 \text{ liters} \end{aligned}$$

Division of a denominated number by a denominated number is usually performed in square units divided by a value denominated in the same units.

Example:

$$20 \text{ m}^2 / 2 \text{ m} = 10 \text{ m}.$$

AREA AND VOLUME

Area is a measurement of the amount of surface of an object. Also, the space inside the borders of a geometric construction is the area of that figure. Area is expressed in square units. For small objects, square centimeters is a common denomination. Larger objects may use square meters or square kilometers.

Mensuration is the measuring of geometric magnitudes, lengths, areas and volumes. Specific formulas are used to calculate the areas of various geometric shapes. In the following paragraphs, common geometric shapes are defined and the formula for calculating the area of each are given. A summary of shapes and area formulas is shown in **Figure 1-15**.

OBJECT	AREA	FORMULA
Rectangle	Length \times Width	$A = LW$
Square	Length \times Width	$A = LW$
	Side \times Side	$A = S^2$
Triangle	$\frac{1}{2} \times (\text{Base} \times \text{Height})$	$A = \frac{1}{2} BH$
Parallelogram	Length \times Height	$A = LH$
Trapezoid	$\frac{1}{2} \times (\text{base}_1 + \text{base}_2) \times \text{Height}$	$A = \frac{1}{2} (b_1 + b_2)H$
Circle	$\pi \times \text{radius}^2$	$A = \pi r^2$
Ellipse	$\pi \times \text{semi-axis } a \times \text{semi-axis } b$	$A = \pi ab$

Figure 1-15. Formulas to compute areas of common geometric constructions.

RECTANGLE

A *rectangle* is a 4-sided figure with opposite sides that are equal in length and parallel to each other. All of the angles of a rectangle are right angles and the sum of all of the angles is 360° . (*A right angle is a 90° angle.*) The rectangle is a very familiar shape in aircraft sheet metal fabrication. (**Figure 1-16**) The formula for the area of a rectangle is:

$$\text{Area} = \text{Length} \times \text{Width} \text{ or } A = L \times W$$

To calculate the area of a rectangle, determine the length and width and multiply them together.

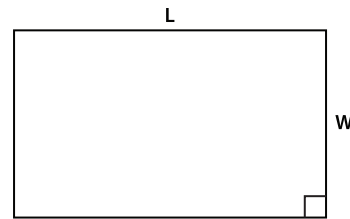


Figure 1-16. A rectangle.

Example:

A rectangular aircraft floor panel has a length of 24 centimeters and a width of 12 centimeters. What is the area of the floor panel expressed in square centimeters?

$$A = L \times W, \quad A = 24 \text{ cm} \times 12 \text{ cm}, \quad A = 288 \text{ cm}^2$$

SQUARE

A *square* is a 4-sided figure with all four sides of equal length and opposite sides that are parallel to each other. (**Figure 1-17**) All the angles contained in a square are right angles and the sum of all of the angles is 360° . A square is actually a rectangle with 4 equal sides. Therefore the area of a square is the same as that of a rectangle: Area = Length \times Width or, $A = L \times W$. However, since the sides of a square are always the same value (S), the formula for the area of a square can also be written as follows:

$$\text{Area} = \text{Side} \times \text{Side} \text{ or } A = S^2$$

To calculate the area of a square, determine the length of a side and perform the arithmetic in the formula.

Example:

What is the area of a square access plate whose side measures 25 centimeters?

$$A = S^2, \quad A = 25 \text{ cm} \times 25 \text{ cm}, \quad A = 625 \text{ cm}^2$$

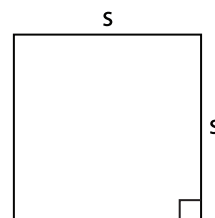


Figure 1-17. A square.

TRIANGLE

A *triangle* is a three-sided figure. The sum of the three angles in a triangle is always equal to 180° . Triangles are often classified by their sides. An *equilateral* triangle has 3 sides of equal length. An *isosceles* triangle has 2 sides of equal length. A *scalene* triangle has three sides of differing length. Triangles can also be classified by their angles: An *acute* triangle has all three angles less than 90° . A *right triangle* has one right angle (a 90° angle). An *obtuse* triangle has one angle greater than 90° . Each of these types of triangles is shown in **Figure 1-18**.

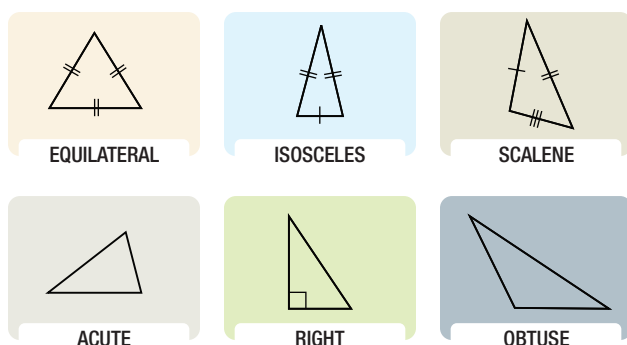


Figure 1-18. Types of triangles.

The formula for the area of a triangle is:

$$\text{Area} = \frac{1}{2} \times (\text{Base} \times \text{Height}) \text{ or } A = \frac{1}{2} BH$$

Example:

Find the area of the right triangle shown in **Figure 1-19**. First, substitute the known values into the area formula.

$$A = \frac{1}{2} (B \times H) = \frac{1}{2} (1.2 \text{ m} \times 750 \text{ cm})$$

Next, convert all dimensions to centimeters (or meters):

$$A = \frac{1}{2} (1\,200 \text{ cm} \times 750 \text{ cm}) \text{ or;} \\ A = \frac{1}{2} (1.2 \text{ m} \times .75 \text{ m})$$

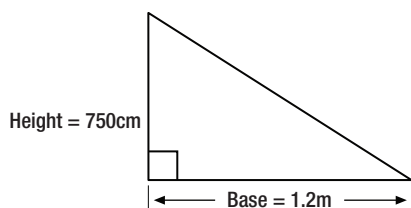


Figure 1-19. An right triangle.

Now, solve the formula for the unknown value:

$$A = \frac{1}{2} (900\,000 \text{ cm}^2) \text{ or } A = \frac{1}{2} (.9 \text{ m}^2) \\ A = 450\,000 \text{ cm}^2 \quad A = .45 \text{ m}^2$$

PARALLELOGRAM

A *parallelogram* is a four-sided figure with two pairs of parallel sides. (**Figure 1-20**) Parallelograms do not necessarily have four right angles like rectangles. However, the sum of the angles in a parallelogram is 360° . Similar to a rectangle, the formula for the area of a parallelogram is:

$$\text{Area} = \text{Length} \times \text{Height} \quad A = LH$$

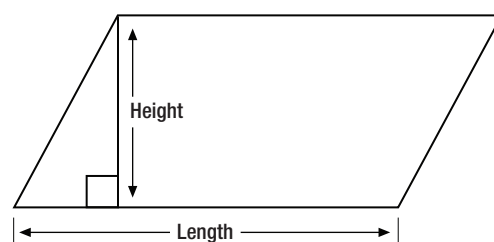


Figure 1-20. A Parallelogram.

To find the area of a parallelogram, simply substitute values into the formula or multiply the length times the height.

TRAPEZOID

A *trapezoid* is a four-sided figure with one pair of parallel sides known as base_1 and base_2 and a height which is the perpendicular distance between the bases. (**Figure 1-21**) The sum of the angles in a trapezoid is 360° . The formula for the area of a trapezoid is:

$$\text{Area} = \frac{1}{2} (\text{Base}_1 + \text{Base}_2) \times \text{Height}$$

Example:

What is the area of the trapezoid in **Figure 1-22** whose bases are 35 centimeters and 25 centimeters, and whose height is 15 centimeters?

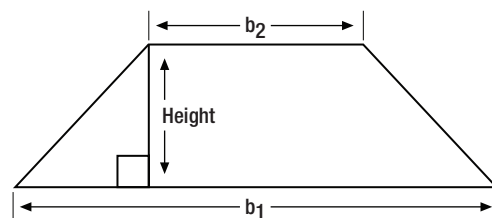


Figure 1-21. A trapezoid has 1 set of parallel sides known as base_1 and base_2 and a height which is the perpendicular distance between the bases.

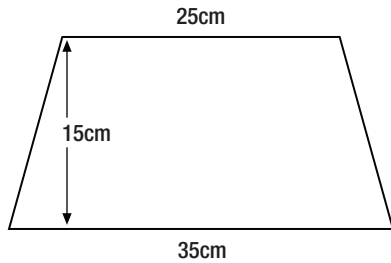


Figure 1-22. A trapezoid with dimensions.

Substitute the known values into the formula and perform the arithmetic.

$$\begin{aligned}
 A &= \frac{1}{2} (b_1 + b_2) \times H \\
 A &= \frac{1}{2} (35 \text{ cm} + 25 \text{ cm}) \times 15 \text{ cm} \\
 A &= \frac{1}{2} (60 \text{ cm}) \times 15 \text{ cm} \\
 A &= 450 \text{ cm}^2
 \end{aligned}$$

CIRCLE

A *circle* is a closed, curved, plane figure. (**Figure 1-23**) Every point on the circle is an equal distance from the center of the circle. The *diameter* is the distance across the circle (*through the center*). The *radius* is the distance from the center to the edge of the circle. The diameter is always twice the length of the radius. The *circumference* of a circle, or distance around a circle is equal to the diameter times π (3.141 6).

Written as a formula:

$$\text{Circumference} = \pi \times d \quad \text{or} \quad C = 2 \pi \times r$$

The formula for finding the area of a circle is:

$$\text{Area} = \pi \times \text{radius}^2 \quad \text{or} \quad A = \pi r^2$$

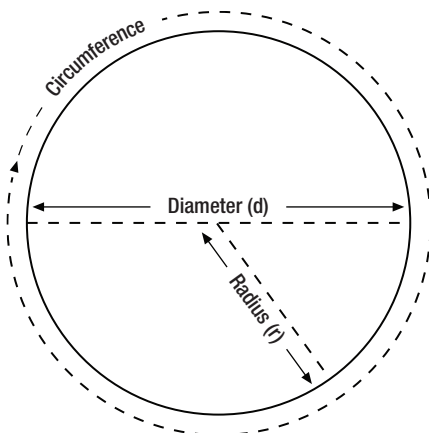


Figure 1-23. A circle.

Example:

The bore, or "inside diameter," of a certain aircraft engine cylinder is 12 centimeters. Find the area of the cross section of the cylinder.

First, substitute the known values into the formula:

$$A = \pi r^2 = 3.141 \, 6 \times (1\frac{1}{2} \text{ cm})^2$$

Note that the diameter is given but since the diameter is always twice the radius, dividing the diameter by 2 gives the dimension of the radius (6 cm). Now perform the arithmetic:

$$\begin{aligned}
 A &= 3.141 \, 6 \times 36 \text{ cm}^2 \\
 A &= 113.097 \, 6 \text{ cm}^2
 \end{aligned}$$

Example:

A cockpit instrument gauge has a round face that is 3 inches in diameter. What is the area of the face of the gauge? From **Figure 1-11** for $N = 3$, the answer is 7.068 6 square inches. This is calculated by: If the diameter of the gauge is 3 inches, then the radius = $\frac{3}{2} = 1\frac{1}{2}$ inches.

$$\begin{aligned}
 \text{Area} &= \pi \times r^2 = 3.141 \, 6 \times 1.5^2 = 3.141 \, 6 \times 2.25 \\
 &= 7.068 \, 6 \text{ square inches.}
 \end{aligned}$$

ELLIPSE

An *ellipse* is a closed, curved, plane figure and is commonly called an *oval*. (**Figure 1-24**)

In a radial engine, the articulating rods connect to the hub by pins, which travel in the pattern of an ellipse (*i.e., an elliptical path*). The formulas for the circumference and area of an ellipse are given in **Figure 1-24**.

WING AREA

Wing surface area is important to aircraft performance. There are many different shapes of wings. To calculate wing area exactly requires precise dimensions for the clearly defined geometric area of the wing. However, a general formula for many wing shapes that can be described using an average wing "chord" dimension is similar to the area of a rectangle. The wingspan, S , is the length of the wing from wingtip to wingtip.

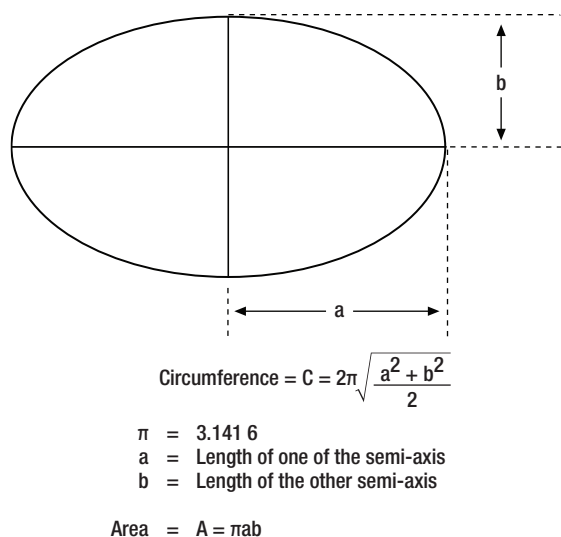


Figure 1-24. An ellipse with formulas for calculating circumference and area.

The chord (C) is the average or mean width of the wing from leading edge to trailing edge as shown in **Figure 1-25**.

The formula for calculating wing area is:

$$\text{Area of a Wing} = \text{Wing Span} \times \text{Mean Chord} \text{ or } AW = SC$$

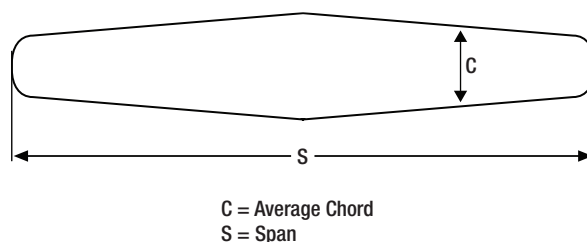


Figure 1-25. Area of an aircraft wing.

Example:

Find the area of a tapered wing whose span is 15 meters and whose mean chord is 2 meters. As always, substitute the known values into the formula.

$$AW = SC$$

$$AW = 15 \text{ meters} \times 2 \text{ meters}$$

$$AW = 30 \text{ square meters (30 m}^2\text{)}$$

VOLUME

Three-dimensional objects have length, width, and height. The most common three dimensional objects are *rectangular solids*, cubes, cylinders, spheres, and cones. *Volume* is the amount of space within an object. Volume is expressed in cubic units. Cubic centimeters are typically used for small spaces and cubic meters for larger spaces, however any distance measuring unit can be employed if appropriate. A summary of common three-dimensional geometric shapes and the formulas used to calculate their volumes is shown in **Figure 1-26**.

OBJECT	VOLUME
Rectangular Solid	LWH
Cube	S^3
Cylinder	$\pi r^2 H$
Sphere	$\frac{4}{3}\pi r^3$
Cone	$\frac{1}{3}\pi r^2 H$

Figure 1-26. Formulas to compute volumes of common geometric three-dimensional objects.

RECTANGULAR SOLIDS

A rectangular solid is any three-dimensional solid with six rectangle-shaped sides. (**Figure 1-27**)

The volume is the number of cubic units within the rectangular solid. The formula for the volume of a rectangular solid is:

$$\text{Volume} = \text{Length} \times \text{Width} \times \text{Height} \text{ or } V = LWH$$

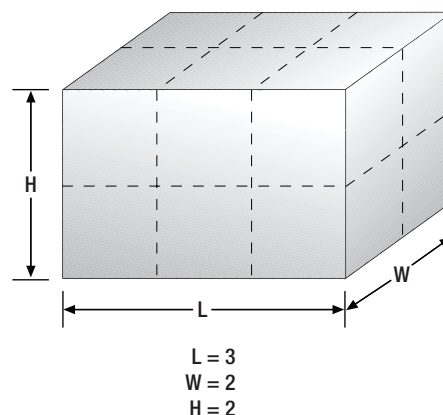


Figure 1-27. A rectangular solid.

Example:

A rectangular baggage compartment measures 2 meters in length, 1.5 meters in width, and 1 meter in height. How many cubic meters of baggage will it hold?

Substitute the known values into the formula and perform the arithmetic.

$$\begin{aligned}V &= LWH \\V &= 2\text{ m} \times 1.5\text{ m} \times 1\text{ m} \\V &= 3\text{ m}^3 \quad \text{or} \quad V = 3\text{ cubic meters}\end{aligned}$$

CUBE

A *cube* is a solid with six square sides. (**Figure 1-28**) A cube is just a special type of rectangular solid. It has the same formula for volume as does the rectangular solid which is Volume = Length \times Width \times Height = $L \times W \times H$. Because all of the sides of a cube are equal, the volume formula for a cube can also be written as:

$$\text{Volume} = \text{Side} \times \text{Side} \times \text{Side} \quad \text{or} \quad V = S^3$$

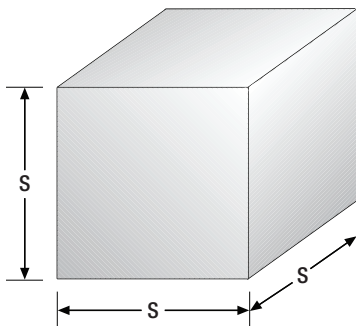


Figure 1-28. A cube.

Example:

A cube-shaped carton contains a shipment of smaller boxes inside of it. Each of the smaller boxes is 10 cm \times 10 cm \times 10 cm. The measurement of the large carton is 30 cm \times 30 cm \times 30 cm. How many of the smaller boxes are in the large carton?

Substitute the known values into the formula and perform the arithmetic:

$$\begin{aligned}\text{Large Box} \\V &= L \times W \times H \\V &= 30\text{ cm} \times 30\text{ cm} \times 30\text{ cm} \\V &= 27\,000\text{ cubic centimeters of volume in large carton}\end{aligned}$$

Small Box

$$\begin{aligned}V &= L \times W \times H \\V &= 10\text{ cm} \times 10\text{ cm} \times 10\text{ cm} \\V &= 1\,000\text{ cubic centimeters of volume in small cartons.}\end{aligned}$$

Therefore, since each of the smaller boxes has a volume of 1 000 cubic centimeters, the large carton will hold 27 boxes (27 000 \div 1 000).

Substitute the known values into the formula and perform the arithmetic:

Large Box

$$\begin{aligned}V &= S^3 \\V &= 30\text{ cm} \times 30\text{ cm} \times 30\text{ cm} \\V &= 27\,000\text{ cubic centimeters of volume in large carton.}\end{aligned}$$

Small Box

$$\begin{aligned}V &= S^3 \\V &= 10\text{ cm} \times 10\text{ cm} \times 10\text{ cm} \\V &= 1\,000\text{ cubic centimeters of volume in small cartons.}\end{aligned}$$

Therefore, since each of the smaller boxes has a volume of 1 000 cubic centimeters, the large carton will hold 27 boxes (27 000 \div 1 000).

CYLINDER

A *cylinder* is a hollow or solid object with parallel sides the ends of which are identical circles. (**Figure 1-29**)

The formula for the volume of a cylinder is:

$$\begin{aligned}\text{Volume} &= \pi \times \text{radius}^2 \times \text{height of the cylinder} \\ \text{or, } V &= \pi r^2 H\end{aligned}$$

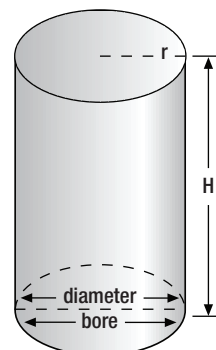


Figure 1-29. A cylinder.

One of the most important applications of the volume of a cylinder is finding the *piston displacement* of a cylinder in a reciprocating engine. Piston displacement is the total volume (in cubic inches, cubic centimeters, or liters) swept by all of the pistons of a reciprocating engine as they move during one revolution of the crankshaft. The formula for piston displacement is given as:

$$\text{Piston Displacement} = \pi \times (\text{bore divided by } 2)^2 \times \text{stroke} \times (\# \text{ cylinders})$$

The bore of an engine is the inside diameter of the cylinder. The stroke of an engine is the length the piston travels inside the cylinder. (**Figure 1-30**)

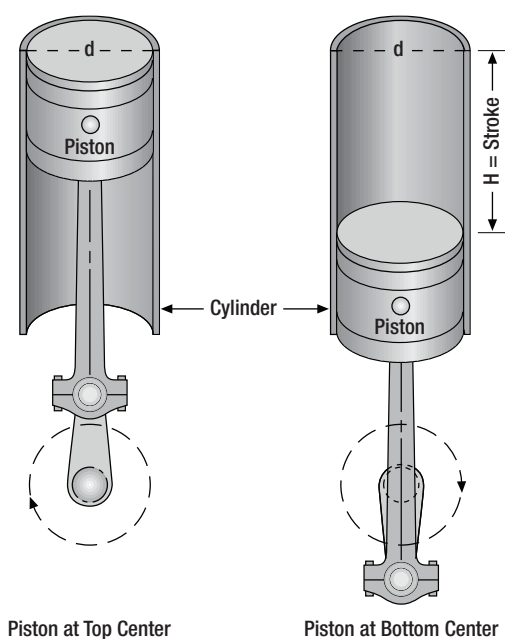


Figure 1-30. Piston displacement in an engine cylinder.

Example:

Find the piston displacement of one cylinder in a multi-cylinder aircraft engine. The engine has a cylinder bore of 13.97 centimeters and a stroke of 13.716 centimeters. First, substitute the known values in to the cylinder formula.

$$\begin{aligned} V &= \pi r^2 h = (3.1416) \times (13.97 \text{ cm} \div 2)^2 \times (13.716 \text{ cm}) \\ V &= (3.1416) \times (6.985 \text{ cm})^2 \times 13.716 \text{ cm} \\ &= 2102.3798 \text{ cm}^3 \end{aligned}$$

Piston displacement of one cylinder is 2102.3798 cubic centimeters.

For a 6-cylinder engine, then the total engine displacement would be:

$$\begin{aligned} \text{Total Displacement for 6 cylinders} &= \\ 6 \times 2102.3798 \text{ cm}^3 &= 12614.2788 \\ \text{cubic centimeters of displacement.} \end{aligned}$$

SPHERE

A solid having the shape of a ball is called a *sphere*. (**Figure 1-31**) A sphere has a constant diameter. The radius (r) of a sphere is one-half of the diameter (D). The formula for the volume of a sphere is:

$$\text{Volume} = \frac{4}{3} \times \pi \times \text{radius}^3 \text{ or } V = \frac{4}{3} \times \pi r^3$$

Example:

A pressure tank inside the fuselage of a cargo aircraft is in the shape of a sphere with a diameter of 80 centimeters. What is the volume of the pressure tank?

$$\begin{aligned} V &= \frac{4}{3} \pi r^3 = \frac{4}{3} \times (3.1416) \times \left(\frac{80 \text{ cm}}{2}\right)^3 \\ &= 1.33 \times 3.1416 \times (40 \text{ cm})^3 \\ &= 1.33 \times 3.1416 \times 64000 \text{ cm}^3 \\ V &= 267412.99 \text{ cubic centimeters} \end{aligned}$$

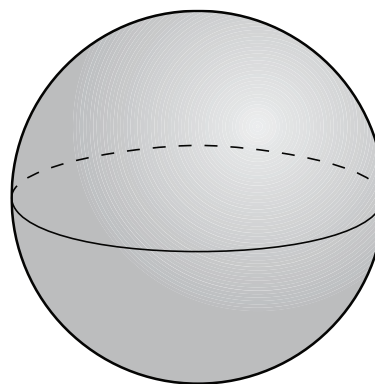


Figure 1-31. A sphere.

CONE

A solid or hollow object with a circle as a base and with sides that gradually taper to a point is called a *cone*. (**Figure 1-32**)

The formula for the volume of a cone is:

$$V = \frac{1}{3} \pi r^2 H$$

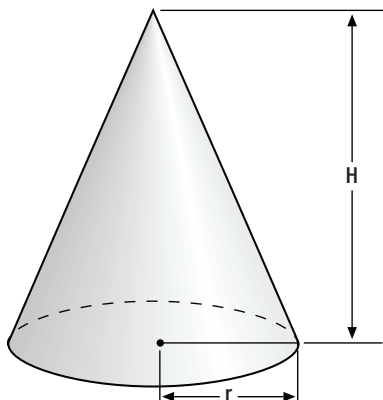


Figure 1-32. A cone.

To find the volume of a cone, determine the radius and height, substitute the values into the formula and perform the arithmetic. (*Figure 1-26*)

WEIGHTS AND MEASURES

In nearly every country, a system of weights and measures is in place. Often, it has been handed down from generation to generation derived from localized customs of how various items are measured by society. In 1870, international standardization was born. A meeting of 15 countries in Paris ultimately led to the establishment of a permanent International Bureau of Weights and Measures in 1875. A General Conference on Weights and Measures was constituted to handle all international matters concerning the metric system. The first meeting was in 1889 at which the kilogram and the meter were legalized as the international units of weight and length. Over the years, an increasingly larger group of countries have met periodically at this conference to ensure worldwide uniformity in units of measurement. In 1960, the current name, International System of Units (SI) was adopted. New units of measurement have been agreed upon and refinements made at periodic meeting since that time.

The International Organization for Standardization (ISO) is a worldwide federation of national standards institutes from numerous countries. It provides recommendations for the use of SI and certain other units. The ISO publishes documents that provide extensive detail on the application of SI units. ICAO, the International Civil Aviation Organization, maintains liaison with the ISO regarding the standardized

application of SI units in aviation. ICAO publishes ANNEX 5 - Units of Measurement to be Used in Air and Ground Operations to guide aviation organizations and individuals in the use of SI, the standardized system of weights and measures in aviation.

Effort was made in the assembly of this series to incorporate SI standards and practices as guided by ICAO. There are some exceptions. Should the reader encounter non-SI units of weights and measurements, it is owing to the continued widespread use of certain non-SI units in aviation or the learning value of presenting examples in non-SI units.

For most people, there are two basic systems of weights and measure: the Imperial System and the SI/Metric System. The Imperial System is rooted in Great Britain and is the dominant system of weights and measure in the United States of America. It uses widely familiar units that for years have dominated aviation such as feet for distance and pounds for weight. The other system is the metric system which has a strong foothold in Europe and other regions around the world. It uses meters and kilometers for distance measuring units and kilograms for weight units. The metric system is formalized in SI (the International System of Units) which is the official system of weights and measure of ICAO, the International Civil Aviation Organization.

As a major aviation force and dominant manufacturer of civil aircraft, the United States continues to use the Imperial System with gradual integration of SI over recent decades. Aviators in most countries around the world are familiar with some units from the Imperial System and SI with a tendency towards common use of one or the other. It is important to be able to convert values from one system to another to ensure the accurate incorporation of critical data for aviation operations. In the reference pages at the end of this module, several tables exist. Some define the units of measurement standards incorporated in ANNEX 5 for world aviation standardization. Other tables provide conversion between the Imperial System and SI. Many conversion factors are also listed. These are numerical values that can be used to multiply or divide a value in one system to give the value in the other system. A sample of a few conversion values are listed in *Figure 1-33*.

COMMON CONVERSIONS					
IMPERIAL SYSTEM			SI (METRIC)		
<i>Distance</i>					
1 Inch	is equal to		2.54 Centimeters		
1 Foot	is equal to		0.304 8 Meters		
1 (Statute) Mile	is equal to		1.609 3 Kilometers		
<i>Weight</i>					
1 Pound	is equal to		0.453 59 Kilograms		
<i>Volume</i>					
1 Quart	is equal to		0.946 35 Liters		
1 Gallon	is equal to		3.785 4 Liters		
<i>Temperature</i>					
0 Fahrenheit	is equal to		(-)17.778 Celsius		
0 Fahrenheit	is equal to		255.37 Kelvin		
<i>Area</i>					
1 Square Inch	is equal to		6.451 6 Square Centimeters		
1 Square Foot	is equal to		0.092 903 Square Meters		
1 Square Mile	is equal to		2.59 Square Kilometers		
<i>Speed</i>					
1 Foot Per Second	is equal to		0.304 8 Meters Per Second		
1 Mile Per Hour	is equal to		1.609 3 Kilometers Per Hour		
1 Knot	is equal to		1.852 Kilometers Per Hour		
			SI (METRIC)		
			<i>Distance</i>		
			1 Centimeter	is equal to	0.394 Inches
			1 Meter	is equal to	3.28 Feet
			1 Kilometer	is equal to	0.621 Miles
			<i>Weight</i>		
			1 Kilogram	is equal to	2.204 Pounds
			<i>Volume</i>		
			1 Liter	is equal to	1.057 Quarts
			1 Lites	is equal to	0.264 Gallons
			<i>Temperature</i>		
			0 Celsius	is equal to	33.8 Fahrenheit
			0 Kelvin	is equal to	(-)437.87 Fahrenheit
			<i>Area</i>		
			1 Square Centimeter	is equal to	0.155 Square Inches
			1 Square Meter	is equal to	10.763 91 Square Feet
			1 Square Kilometer	is equal to	0.386 Square Miles
			<i>Speed</i>		
			1 Meters Per Second	is equal to	3.281 Feet Per Second
			1 Kilometer Per Hour	is equal to	0.621 Miles Per Hour
			1 Kilometer Per Hour	is equal to	0.540 Knots

Figure 1-33. Common conversions chart.

Question: 1-1

An airplane is flying at 22 000 feet above sea level. The current temperature at sea level is -4° and the temperature at 22 000 feet above sea level is 25° cooler. If the ratio of temperature change remains constant as altitude is increased, what is the temperature at 30 000 feet?

- A. -21°
- B. -29°
- C. -34

Question: 1-2

When working with fractions, the common denominator is found by multiplying all of the denominators together.

- A. True
- B. False
- C. Cannot be determined

Question: 1-3

Some examples of units used in the Imperial measurement system include:

- A. inch, foot, yard.
- B. inch, meter, rod.
- C. inch, kilometer, fraction.

Question: 1-4

What is the speed of a spur gear with 42 teeth driven by a pinion gear with 14 teeth turning 420 RPM?

- A. 588 RPM
- B. 160 RPM
- C. 140 RPM

Question: 1-5

What is the definition of proportion?

- A. A statement of equality between two or more ratios.
- B. A statement of inequity between two or more ratios.
- C. All ratios are proportionate.

Question: 1-6

A pilot flies .81 of the way to her destination before she stops and refuels. How much is left of her trip?

- A. cannot be determined
- B. 19 miles
- C. 19%

Question: 1-7

What power of 10 is equal to 1 000 000?

- A. 10 to the fourth power
- B. 10 to the fifth power
- C. 10 to the sixth power

Question: 1-8

The result of 7 raised to the third power plus the square root of 39 is equal to:

- A. 349.24
- B. .34924
- C. 343.24

ANSWERS

Answer: 1-1

Answer is C.

Take the 25° C difference in temperature and divide it by 22 000 ($25/22\ 000 = .001\ 136\ 4$), which gives us the degree change per foot. Now we know we are increasing by 8 000 foot $.001\ 136\ 4 \times 8\ 000 = 9.09^\circ$ degrees. The best answer is -34° . If the temperature change between 0 and 22 000 feet is 25°, dividing 25 000 by 22 000 tells you that for each 1 000 feet of altitude gained, the temperature decreases 1.136°. 1.136×30 (30 000 feet) = 34.08° , meaning that the temperature at 30 000 feet is approximately 34° degrees less than at sea level. -4 minus $34 = -38^\circ$ at 30 000 feet.

Answer: 1-2

Answer is A.

Mathematical principle states that when adding and subtracting fractions the fractions must have a common denominator. A simple way to find the common denominator is to multiply all the denominators together. Note, however, that this number will not always be the least common denominator (LCD) but can be used to continue the calculations. When the final fraction is determined, it may have to be reduced to its lowest terms and/or corrected so it is not an improper fraction.

Answer: 1-3

Answer is A.

The meter and kilometer are metric measurements and a fraction is a mathematical unit not a unit of measure.

Answer: 1-4

Answer is C.

The ratio of the two gears is 42:14 or simplified to 3:1. Since the driving gear (pinion gear) is smaller than the gear it is turning (spur gear), the speed of the spur gear must also be less. Thus to find the speed of the spur gear, divide the pinion gear speed by 3. 3.420 divided by $3 = 1.140$.

Answer: 1-5

Answer is A.

A proportion is a statement of equality between two or more ratios. For example, A is to B as C is to D can be represented as $A:B = C:D$ or $A/B = C/D$.

Answer: 1-6

Answer is C.

This is percentage question. .81 is equivalent to 81%. Subtracting 81% from 100% equals 19%. Read the whole question and all the answers. Although percentages are defined as "parts of one hundred", 19% of 1 200-mile trip is 228 miles.

Answer: 1-7

Answer is C.

The power (or exponent) of a number is a shorthand method of indicating how many times a number, called the base, is multiplied by itself. Ten to the sixth power or 10^6 equals $10 \times 10 \times 10 \times 10 \times 10 \times 10$ equals 1 000 000. For a base number of 10 simply add zeros to equal the power, e.g. $10^2 = 100$, $10^4 = 10\ 000$, etc.

Answer: 1-8

Answer is A.

$$7^3 = 7 \times 7 \times 7 = 343$$

$$\text{Square root of } 39 \text{ is } 6.24$$

$$343 + 6.24 = 349.24$$

Question: 1-9

Find the square root of 124.9924.

- A. 111.8×10 to the third power.
- B. $.111\ 8 \times 10$ to the negative second power.
- C. $1\ 118 \times 10$ to the negative second power.

Question: 1-10

What is the cube root of 216?

- A. 10 077 696
- B. 72
- C. 6

Question: 1-11

The number 3.47×10 to the negative fourth power is equal to:

- A. .003 47
- B. 34 700.0
- C. .000 347

Question: 1-12

Which of the following numbers is written using Engineering Format?

- A. 12.34×10^3
- B. 1.234×10^5
- C. 0.1234×10^7

Question: 1-13

The value of $\frac{1}{6} + \frac{4}{6} \times \frac{5}{4}$ is:

- A. 1
- B. $\frac{5}{36}$
- C. $\frac{25}{24}$

Question: 1-14

A box has dimensions of 30mm, 10cm, and 40mm. What is its volume?

- A. 1 200 cm³
- B. 120 cm³
- C. 12 cm³

Question: 1-15

Given that 1 liter = 1 000 cm³

The base of a rectangular tank is 150 cm by 120 cm. If the tank holds 36 liters of water, then its height in cm is:

- A. 20 cm
- B. 200 cm
- C. 2 cm

Question: 1-16

Convert 386.27 MHz to kHz

- A. $3\ 862 \times 10^3$ kHz
- B. 386.27×10^6 kHz
- C. 386.27×10^3 kHz

ANSWERS

Answer: 1-9

Answer is C.

The square root of 124.992 4 is 11.18, which is equivalent to $1\ 118 \times 10^{-2}$.

Answer: 1-13

Answer is A.

Multiply before adding, so $\frac{4}{6} \times \frac{5}{4} = \frac{20}{24} = \frac{5}{6}$

then add $\frac{1}{6} + \frac{5}{6} = 1$

Answer: 1-10

Answer is C.

When 6 is multiplied by itself 3 times it equals 216 ($6 \times 6 \times 6$). The other two answers are incorrect.

$72 = 216/3$ and 10 077 696 is actually 216^3 or

$216 \times 216 \times 216$.

Answer: 1-14

Answer is D.

Change mm to cm, 30 mm = 3 cm, 40 mm = 4 cm

Then volume = $3 \times 10 \times 4 = 120 \text{ cm}^3$

Answer: 1-11

Answer is C.

Use the reciprocal operation. $1/3.47 \times 10_4$, which is $1/34\ 700$ or 0.000 347.

Answer: 1-15

Answer is C.

$$150 \times 120 \times \text{height} = 36\ 000$$

$$18\ 000 \times \text{height} = 36\ 000$$

$$\text{height} = 2 \text{ cm}$$

Answer: 1-12

Answer is A.

If a number is written using Engineering Format, the power of 10 is a multiple of 3.

The answer is 12.34×10^3 .

Answer: 1-16

Answer is C.

$$386.27 \text{ MHz} = 386.27 \times 10^6 \text{ Hz}$$

$$386.27 \times 10^6 \text{ Hz} = 386.27 \times 10^3 \times 10^3 \text{ Hz}$$

$$386.27 \times 10^3 \times 10^3 \text{ Hz} = 386.27 \times 10^3 \text{ kHz}$$



MATHEMATICS

ALGEBRA

SUB-MODULE 02

PART-66 SYLLABUS LEVELS

CERTIFICATION CATEGORY →

B1

B2

Sub-Module 02

ALGEBRA

Knowledge Requirements

2.1 - Algebra

- (a) Evaluating simple algebraic expressions, addition, subtraction, multiplication and division, use of brackets, simple algebraic fractions.
- (b) Linear equations and their solutions;
Indices and powers, negative and fractional indices;
Binary and other applicable numbering systems;
Simultaneous equations and second degree equations with one unknown;
Logarithms.

2

2

1

1

Level 1

A familiarization with the principal elements of the subject.

Objectives:

- (a) The applicant should be familiar with the basic elements of the subject.
- (b) The applicant should be able to give a simple description of the whole subject, using common words and examples.
- (c) The applicant should be able to use typical terms.

Level 2

A general knowledge of the theoretical and practical aspects of the subject and an ability to apply that knowledge.

Objectives:

- (a) The applicant should be able to understand the theoretical fundamentals of the subject.
- (b) The applicant should be able to give a general description of the subject using, as appropriate, typical examples.
- (c) The applicant should be able to use mathematical formula in conjunction with physical laws describing the subject.
- (d) The applicant should be able to read and understand sketches, drawings and schematics describing the subject.
- (e) The applicant should be able to apply his knowledge in a practical manner using detailed procedures.

ALGEBRA IN AVIATION MAINTENANCE

In this sub-module, the basic concepts of algebra are introduced. Evaluation of simple algebraic expressions by correct manipulation of algebraic terms and simple algebraic fractions is covered here.

Algebraic equations are also explored. Methods of finding solutions to linear and quadratic equations are given. An introduction to simultaneous equations is

included. Indices and powers using algebraic terms are reviewed and logarithms are presented as a natural application when working with number indices.

The binary, octal and hexadecimal number systems are included in relation to the decimal number system and the application of the binary number system in modern electronics is introduced.

ALGEBRA

Algebra is the next step in mathematics. Arithmetic uses numbers. *Algebra* uses letters or symbols to represent numbers and show relationships between them. Algebra extends the concepts of arithmetic so that it is possible to generalize the rules for operating with numbers. In algebra, formulas and equations are written with numbers, letters and symbols. Known number values are substituted into the formulas and equations in place of the letters and symbols. Then, arithmetic is used to find unknown values.

A *variable* is the name given to a value or number that is not yet known. It can change depending on the situation being considered. While the numerical values of variables can change, the relationship between the variables in a formula or equation remains unchanged. For example, to find the distance an aircraft travels, a formula is used. It is also an equation. The distance an aircraft travels can be calculated if its velocity and duration of flight are known. The relationship between the displacement, velocity and time can be expressed in the following formula:

$$\text{Displacement} = \text{Velocity} \times \text{Time}$$

This relationship between the variables Displacement, Velocity and Time never changes. But numerical values for each variable can change depending upon the flight being considered. Instead of writing out words for the variables in algebra, letters are used to represent words. The letter *s*, is normally used for displacement, *v* for velocity and *t* for time. So the formula above is expressed as follows:

$$s = v \times t \text{ or } v = \frac{s}{t}$$

Note when two letters or when a letter, number or symbol are written next to each other without a space between them, it is the same as if there is a multiplication symbol between them:

$$ab \text{ is the same as } a \times b \quad 4y \text{ is the same as } 4 \times y \\ a(b + 7) \text{ is the same as } a \times (b + 7)$$

To find the distance traveled on a flight, substitute known values in place of the letters in the formula and do the arithmetic. If the velocity of an aircraft is 220 kilometers per hour and the aircraft has been flying for 2 hours, the displacement is calculated as follows:

$$s = v \times t = 220 \times 2 = 440 \text{ km}$$

Don't forget to include units, in this case meters, when answering this type of question.

As stated, in algebra, formulas and equations are written using letters and symbols to show the relationship between variables. A *term* in algebra is any letter, number or combination of letters and numbers that is used in an algebraic expression.

For example, in the expression $v + 4x - 3y + (a - c)$ there are 4 terms, namely *v*, *4x*, *3y* and $(a - c)$

In algebra, like terms have the same value. If a variable is described as *x*, then every time *x* appears in an expression or equation, it will have the same value.

When a number appears on its own in algebra, it is known as a *constant*.

When a number appears in front of a letter or variable, it is known as a *coefficient*. In the terms listed above, 4 is the coefficient of x and 3 is the coefficient of y .

An *expression* in algebra is a term or a group of terms separated by one or more operators: $+$, \div , \times , or $-$ signs.

EVALUATING SIMPLE ALGEBRAIC EXPRESSIONS

This topic was introduced in *sub-module 01*, we expand on it in this section. Equations in algebra can be manipulated or solved, often to learn the value of a particular unknown variable. When solving algebraic equations, the rules described for adding, subtracting, multiplying, and dividing whole numbers and fractions apply. In fact, algebra is a way of generalizing what happens to numbers when they are added, subtracted, multiplied or divided.

ADDITION

As with numbers, when adding algebraic terms, a "+" sign is used between the terms. The result or sum is expressed with an "=" sign. This is shown in the following example:

$$a + b = c$$

An example using numbers is: $1 + 2 = 3$

Addition is *commutative*. This means that the order in which terms are added does not change the sum. The following illustrates the *commutative property of addition*:

$$a + b = b + a$$

Using numbers: $1 + 2 = 2 + 1$

Addition is also *associative*. When more than two terms are added, the sum is the same regardless of the order in which the terms are added. The following illustrates the *associative property of addition*. Brackets are used to define which terms are added first before adding the third term:

$$(a + b) + c = a + (b + c)$$

Using numbers: $(1 + 2) + 3 = 1 + (2 + 3)$

Completing the addition in the brackets first, this becomes:

$$3 + 3 = 1 + 5 \quad 6 = 6$$

Therefore, it doesn't matter in which order the numbers are added, the sum will always equal 6. The same is true for any algebraic expression. The sum of $a + b + c$ is the same regardless of which order the addition is performed.

When adding algebraic terms, only like terms can be combined. For example $2x + 3x = 5x$. However, $2x + 3y$ cannot be combined. It is in its simplest form.

The x in $2x$ represents a variable and the y in $3y$ represents a completely different variable, which cannot be combined with x .

SUBTRACTION

As with numbers, when subtracting algebraic terms, a "-" sign is used between the terms. The result or difference is expressed with an "=" sign:

$$c - b = a$$

An example using numbers is: $3 - 2 = 1$

Subtraction is not commutative. It definitely matters which term is subtracted from the other. $x - y$ may not have the same difference as $y - x$. By substituting numbers for the letters x and y , we can show how this is true:

If $x = 5$ and $y = 3$, then,

$$x - y = 2, \quad (5 - 3) = 2 \quad \text{but} \quad y - x = -2, \quad (3 - 5) = -2$$

To make subtraction more flexible, subtraction of terms can be thought of as addition of a negative or opposite value. For example, $a - b = c$ can be manipulated so that the subtraction becomes addition of a negative number:

$$a - b = a + (-b) \quad \text{and} \quad a + b - c = a + b + (-c)$$

Thus, the commutative property of addition applies:

$$a - b = (-b) + a$$

If $a = 7$ and $b = 5$, substitution of these values for a and b gives the following result:

$$7 - 5 = (-5) + 7, \quad 7 - 5 = 2 \quad \text{and} \quad (-5) + 7 = 2$$

Similarly, thinking of a term that is being subtracted as the addition of the same term with the opposite sign allows the use of the associative property of addition:

$$(a - b) - c = a + ((-b) + (-c))$$

Regardless of which terms are added first before the third term is added, the result is the same.

If:

$$a = 7, b = 5 \quad \text{and} \quad c = 4$$

Then:

$$(7 - 5) - 4 = -2.$$

Also:

$$7 + ((-5) + (-4)) \quad \text{becomes} \quad 7 + (-9) = -2$$

When subtracting algebraic terms, only like terms can be combined. For example, $5x - 3x = 2x$. However, $5x - 3y$ cannot be combined. It is in its simplest form. The x in $5x$ represents a variable and the y in $3y$ represents a completely different variable which cannot be combined with x .

MULTIPLICATION

As with numbers in arithmetic, when multiplying algebraic numbers, the "×" sign is used between the terms. As always, the product is expressed with an "=" sign.

$$a \times b = c$$

With numbers, this could be $2 \times 3 = 6$

If $a = 2$ and $b = 3$ then $c = 6$

Multiplication is both commutative and associative just like addition. Therefore, the following illustrates the *commutative property of multiplication*:

$$a \times b = b \times a.$$

And the *associative property of multiplication* is illustrated as follows:

$$(a \times b) \times c = a \times (b \times c)$$

Multiplication is also distributive. The following illustrates how a term made up of added variables is multiplied by another term. This is the *distributive property of multiplication*:

$$a \times (b \times c) = ab + ac$$

When multiplying polynomials, each term in a bracketed *polynomial* is multiplied by each term in the polynomial multiplier. Then, any combining of like variables can occur. A simple example follows:

$$(a + b) \times (a - b) \quad \text{also written} \quad (a + b)(a - b) \\ a \times a - a \times b + b \times a - b \times b$$

This is written without the "×" symbol as follows:

$$a^2 - ab + ba - b^2$$

Since multiplication is commutative, $ba = ab$ as follows:

$$a^2 - ab + ab - b^2$$

The like terms, $-ab$ and ab , added together equal 0. Thus the final expression becomes:

$$a^2 - b^2$$

DIVISION

Division in algebra is the same as in arithmetic. A "÷" sign is used between the terms to be divided. The "/" sign is also frequently used. Just as with numbers, in algebra division is neither commutative or associative. The terms to be divided must be defined as dividend and divisor and cannot be rearranged or reversed.

RETAINING THE RELATIONSHIP DURING MANIPULATION

When manipulating variables in algebraic equations, the most important rules, as was seen in arithmetic, involve how to manipulate terms properly to arrive at the correct solution.

To solve for a variable in an equation, the term(s) on each side of the = sign can be added to, subtracted from, multiplied, or divided. However, whatever is done on one side of an equation, must also be done on the other side of the equation to ensure that the relationship between terms on each side of the "=" sign remains the same.

Examples: Solve the following equations for the value N.

$$3N = 21$$

To solve for N, divide both sides by 3.

$$3N \div 3 = 21 \div 3$$

$$N = 7$$

$$N + 7 = 24$$

To solve for N, subtract 7 from both sides.

$$N + 7 - 7 = 24 - 7$$

$$N = 17$$

$$N - 22 = 100$$

To solve for N, add 22 to both sides.

$$N - 22 + 22 = 100 + 22$$

$$N = 122$$

$$\frac{N}{5} = 50$$

To solve for N, multiply both sides by 5.

$$\frac{N}{5} \times 5 = 50 \times 5$$

$$N = 250$$

ADDITION AND SUBTRACTION OF EXPRESSIONS WITH PARENTHESES/BRACKETS

Some algebraic problems involve adding terms enclosed by parentheses. It is possible to remove the parentheses by applying the function sign in front of the parentheses to every variable in the term:

$$(8a - 2b) - (10a + b) = 8a - 2b - 10a - b$$

Regrouping the like terms, this becomes:

$$(8a - 10a) - (2b + b) = -2a - 3b$$

ORDER OF OPERATIONS

Exactly as in the *Sub-Module 01 - Arithmetic*, there is an order of operations that must be followed when solving algebraic equations.

To refresh your memory, the order is as follows:

1. Parentheses (Brackets)
2. Powers and Roots
3. Multiplication and Division
4. Addition and Subtraction

SIMPLE ALGEBRAIC FRACTIONS

In algebra, as with numbers, fractions can exist. The rules for manipulating simple algebraic fractions are the same as the rules for manipulating numeric fractions. Some brief examples appear below.

A simple algebraic fraction can look like:

$$\frac{x}{y} \text{ or } \frac{a+b}{c+d}$$

An algebraic fraction can be any algebraic expression or numeric expression \div any algebraic or numeric expression that does not equal 0.

As with numeric fractions, algebraic fractions are multiplied by separately multiplying their denominators and numerators:

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

If there are numerical coefficients involved, multiplication of the coefficients occurs and like variables can be combined. However, unlike variables cannot be combined:

$$\frac{2a}{5x} \times \frac{4b}{3x} = \frac{2a \times 4b}{5x \times 3x} = \frac{8ab}{15x}$$

Note how in the denominator, the coefficients and the variables combine to make $15x$; but in the numerator, only the coefficients could be combined because the variables were not alike (a and b cannot be combined).

When dividing algebraic fractions, proceed as with numerical fractions. That is, invert the divisor and multiply:

$$\frac{a}{b} \div \frac{x}{y} = \frac{a}{b} \times \frac{y}{x} = \frac{ay}{bx}$$

If there are numerical coefficients involved when dividing, combining and reducing the coefficients is allowed but variables that are not the same cannot be combined:

$$\frac{2a}{3b} \div \frac{2x}{3y} = \frac{2a}{3b} \times \frac{3y}{2x} = \frac{6ay}{6bx} = \frac{ay}{bx}$$

Addition and subtraction of simple algebraic fractions follow the same rules as addition and subtraction of numerical fractions. Most importantly, a common denominator must be found before proceeding to add or subtract the terms.

ALGEBRAIC EQUATIONS

Algebraic equations are equations that involve one or more variables. Algebraic equations are used in aviation to show the relationship between two or more variables. For example, the change in charging pressure of a fire extinguisher when the ambient temperature changes, or the increase in manifold pressure when fuel flow is increased. Aircraft designers take these relationships into account and express them as equations. Maintenance technicians in the field typically use graphs and charts of these equations to obtain data needed for proper aircraft servicing.

Equations contain an equals sign (=):

Example:

We have seen in *sub-module 01* that the formula $A = \pi \times r^2$ shows the relationship between the area of a circle (A) and the length of the radius (r) of the circle. The area of a circle is equal to π (3.141 6) times the radius squared. Therefore, the larger the radius, the larger the area of the circle. Other examples of algebraic equations are:

$$2x + 3y = 7$$

$$s = ut + \frac{1}{2}at^2$$

$$E = \frac{1}{2}mv^2$$

We use algebraic equations in the following way:

- We assign a variable to an unknown quantity or quantities.
- We write a mathematical equation based on the information known or given.
- We use appropriate mathematical techniques to solve the equation.
- We interpret the results in terms of the original situation.

LINEAR EQUATIONS

A *linear equation* is an algebraic equation that forms a straight line when plotted on a graph. In a linear equation the highest power of x is 1.

Some examples of linear equations follow:

$$7 = 2x + 1$$

$$0 = -10x + 4$$

(We will see later when we plot the function, y , of a linear equation, that is a function of the general form $y = mx + c$, where m and c are numbers, then it is always true that m is the value of the slope or gradient of the line and c is the value on the y -axis where the line crosses the y -axis.)

For now, let's look at more examples of linear equations:

$$-3 = 2(x + 1)$$

This is linear because the highest power of x is one.
It can also be written in the form:

$$-3 = 2x + 2$$

All of the following are linear equations:

$$3x - 6 = 3x + 4x - 8$$

$$x - 6 = 3 - 2(x - 4)$$

$$5x - 6 = 3x - 8$$

$$\frac{x + 3}{5} = 6$$

$$\frac{-2x + 4x}{10} = 3(x - 4)$$

EXPANDING BRACKETS

When solving equations in algebra it is often necessary to expand brackets. Expanding brackets is equivalent to removing brackets. When removing brackets be aware that we must always follow the precedence rules.

SINGLE BRACKETS

If we have a number or a variable multiplying bracketed terms, then all terms in the brackets must be multiplied as shown in the following examples.

Example:

Expand $3(x + 4)$

Multiple 3 by both terms to get

$$3(x + 4) = 3x + 12$$

Example:

Expand $4x^2(x + 3)$

Multiply both terms by $4x^2$ to get

$$4x^2(x + 3) = 4x^2x + 12x^2$$

Note $4x^2x = 4xxx = 4x^3$, So

$$4x^2(x + 3) = 4x^3 + 12x^2$$

We say x^3 is x raised to the power of 3.

We will deal with powers in more detail later.

MULTIPLYING TWO BRACKETED TERMS

If we want to multiply two bracketed terms by each other, for example:

$$(x + 3)(x + 4)$$

We must multiply all four terms by each other.

The easy way to do this is to break one bracket and keep the other as shown in the following examples:

Example:

Expand $(x + 3)(x + 4)$

$$(x + 3)(x + 4)$$

We write this as:

$$\begin{aligned} & x(x + 4) + 3(x + 4) \\ &= xx + x4 + 3x + 3 \times 4 \\ &= x^2 + 4x + 3x + 12 \\ &= x^2 + 7x + 12 \end{aligned}$$

Example:

Expand $(2x - 5)(x^2 - 2)$

Breaking one bracket gives:

$$\begin{aligned} & (2x - 5)(x^2 - 2) \\ &= 2x(x^2 - 2) - 5(x^2 - 2) \\ &= 2x^3 - 4x - 5x^2 + 10 \end{aligned}$$

Finally, writing the expression, in order of powers of x gives:

$$2x^3 - 5x^2 - 4x + 10$$

Sometimes expressions may involve fractions.

Example:

Expand $x^2 - x^2y^3(2x^2 + \frac{x}{y})$

We multiply outside of the parenthesis first (Do not be tempted to subtract the x^2 !) to get:

$$x^2 - x^2y^3 \cdot 2x^2 - x^2y^3 \cdot \frac{x}{y} = x^2 - 2x^4y^3 - x^3y^2$$

SOLVING LINEAR EQUATIONS

To solve linear equations, we rearrange the equation so that the unknown variable (usually x) is at one side of the equation and the numbers are at the other.

Examples:

Solve each of the following linear equations.

1. Solve: $2x + 4 = 10$

Subtract 4 from both sides:

$$2x = 6$$

Divide both sides by 2,

$$x = 3$$

2. Solve: $6x + 4x = 4(2x + 4)$

Start by adding similar terms on the left hand side:

$$10x = 4(2x + 4)$$

Multiply both terms in the brackets by 4:

$$10x = 8x + 16$$

Subtract $8x$ from both sides:

$$2x = 16$$

Finally, divide both sides by 2:

$$x = 8$$

3. Solve: $\frac{x+2}{5} = 10$

Multiply both sides by 5:

$$x + 2 = 50$$

Subtract 2 from both sides:

$$x = 48$$

4. Solve the following equation for x : $\frac{4}{x} + 5 = 3$

Subtract 5 from each side:

$$\frac{4}{x} = 3 - 5 \quad \text{then,} \quad \frac{4}{x} = -2$$

Multiply both sides by x :

$$4 = -2x$$

Divide both sides by -2,

$$-2 = x$$

One last example:

5. Solve: $\frac{6x+6}{4} + x = 11$

Subtract x from both sides:

$$\frac{6x+6}{4} = 11 - x$$

Multiply both sides by 4. (Be very careful with this step, ALL of the right hand side must be multiplied by 4 - that means both the 11 and the x .)

$$6x + 6 = 44 - 4x$$

Add $4x$ to both sides and subtract 6 from both sides

$$10x = 38$$

Therefore:

$$x = 3.8$$

QUADRATIC EQUATIONS

We can define a *quadratic equation* as an equation where the highest power of x is 2.

A quadratic equation has the general form:

$$ax^2 + bx + c = 0$$

Where a , b and c are real numbers and a cannot be zero. a and b are called the coefficients of x^2 and x respectively.

Examples of quadratic equations are:

$$x^2 - 6x + 8 = 0$$

$$x^2 + 3x + 1 = 0$$

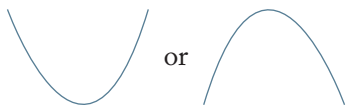
$$16x^2 - 38x - 5 = 0$$

$$4x^2 - 2x = 0$$

We will see later that when we plot a *function*:

$$y = ax^2 + bx + c$$

We will get a graph that is shaped either like:



A quadratic equation in the form:

$$ax^2 + bx + c = 0$$

Can be rewritten as a product of two factors in the form:

$$(?x + ?)(?x + ?) = 0$$

(where the ? represents a number).

FINDING FACTORS

We will look at a number of methods of factorizing quadratic equations.

METHOD 1: USING BRACKETS

The simplest quadratic equations are when the coefficient of x^2 is one, i.e. when $a = 1$. To factorize this type of equation we find two numbers that when multiplied give c , and when added give b .

Example:

$$\text{Factorize } x^2 + 5x + 6 = 0$$

Just a reminder here $a = 1$, $b = 5$ and $c = 6$.

The first step is to write out brackets:

$$(x + ?)(x + ?) = 0$$

To complete the brackets we want two numbers whose product is 6, and whose sum is 5.

The obvious choice is 2 and 3.

The factors are:

$$(x + 2)(x + 3) = 0$$

Note: You can check that these factors are correct by multiplying them out and checking that you get back the original quadratic i.e. $x^2 + 5x + 6 = 0$.

Example:

$$\text{Factorize the quadratic equation } x^2 - x - 12 = 0$$

We find two numbers such that when they are added we get -1 and when multiplied we get -12.

The pairs of numbers:

-1 and 12, -12 and 1, 2 and -6, -2 and 6, 3 and -4, -3 and 4 all multiply to give -12, but the only pair which adds to give -1 is 3 and -4.

Therefore:

$$x^2 - x - 12 = (x - 4)(x + 3) = 0$$

METHOD 2: THE 'BOX METHOD'

This method can be used to factorize quadratic equations when the a is not equal to 1. This method is demonstrated using the following example.

Example: Factorize the quadratic $3x^2 - 10x - 8$

Draw a box with four squares:

Put the x^2 term in the top left:

$3x^2$	

Put the constant term in the bottom right hand corner:

$3x^2$	
	-8

Multiply the ax^2 and c terms to get:

$$(3x^2)(-8) = -24x^2$$

Now we list the factors of $-24x^2$:

$-24x$ and x $24x$ and $-x$
 $-12x$ and $2x$ $12x$ and $-2x$
 $-8x$ and $3x$ $8x$ and $-3x$
 $-6x$ and $4x$ $6x$ and $-4x$

Choose the factors whose sum gives the middle term of the quadratic (in this case $-10x$):

We choose $-12x$ and $2x$

Put these terms in the remaining boxes as shown:

$3x^2$	$-12x$
$2x$	-8

Factor out the common terms to the outside of the box:

	x	-4	
	$3x^2$	$-12x$	$3x$
	$2x$	-8	2

We read the first factor across the top i.e. $(x - 4)$ and read the second factor down i.e. $(3x + 2)$.

We can now write:

$$3x^2 - 10x - 8 = (x - 4)(3x + 2)$$

Some quadratic equations may not have the bx or c term. Examples of these are:

$$4x^2 - 8 = 0 \quad x^2 + 4x = 0 \quad x^2 - 9 = 0$$

We factorize these equations either by factoring out common terms or using the 'Difference of Two Squares'.

METHOD 3: FACTORIZE OUT COMMON TERMS

Example:

Factorize $x^2 + 4x = 0$

x is common to both terms, so:

$$x^2 + 4x = x(x + 4) = 0$$

The factors are simply x and $x + 4$.

METHOD 4: DIFFERENCE OF TWO SQUARES.

Look at the following quadratic equations:

$$x^2 - 4 = 0 \quad x^2 - 9 = 0 \quad x^2 - 25 = 0$$

Each example is the difference of two things squared the first one is the difference of x^2 and 2^2 , the second is the difference of x^2 and 3^2 .

We can write the difference of two squares in the general form:

$$x^2 - y^2$$

The factors of $x^2 - y^2$ are $(x - y)(x + y)$

We can now factorize the examples above:

$$x^2 - 4 = (x + 2)(x - 2) = 0$$

$$x^2 - 9 = (x + 3)(x - 3) = 0$$

$$x^2 - 25 = (x + 5)(x - 5) = 0$$

SOLVING QUADRATIC EQUATIONS

Quadratic equations are solved either by factorizing or by using a formula called the Quadratic Formula.

Quadratic equations will always have **two** solutions (sometimes the two solutions will be the same as each other).

SOLVING USING FACTORS

Example:

Solve: $3x^2 - 10x - 8 = 0$

We factorize as shown above:

$$(x - 4)(3x + 2) = 0$$

We can now write:

$$(x - 4) = 0 \text{ and } (3x + 2) = 0$$

$$\text{If } x - 4 = 0 \text{ then } x = 4$$

$$\text{If } 3x + 2 = 0 \text{ then } 3x = -2 \text{ and } x = -\frac{2}{3}$$

The equation $3x^2 - 10x - 8 = 0$ therefore has two solutions:

$$x = 4 \text{ and } x = -\frac{2}{3}$$

Example:

Solve $x^2 - 9 = 0$

Factorize:

$$(x + 3)(x - 3) = 0$$

Which gives two possibilities:

$$x + 3 = 0 \text{ or } x - 3 = 0$$

and finally two solutions:

$$x = -3 \text{ and } x = 3$$

USING THE QUADRATIC FORMULA

Given:

$$ax^2 + bx + c = 0$$

The values of x are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

Solve the equation.

$$2x^2 - 5x - 3 = 0$$

Solution:

We note that $a = 2$, $b = -5$ and $c = -3$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 + 24}}{4}$$

$$x = \frac{5 \pm \sqrt{49}}{4}$$

$$x = \frac{5 \pm 7}{4}$$

Now we separate the + and – parts:

$$x = \frac{5+7}{4} \quad \text{and} \quad x = \frac{5-7}{4}$$

$$x = \frac{12}{4} \quad \text{and} \quad x = \frac{-2}{4}$$

$$x = 3 \quad \text{and} \quad x = -0.5$$

SIMULTANEOUS EQUATIONS

It is possible to have two (or more) equations that work together. These are called *simultaneous equations*. They are also known as a system of equations. For equations to work together they must share at least one variable. The solution for simultaneous equations that are linear equations is the point on the graph where the solution line for one equation crosses the solution line for the other equation. This represents the *ordered pair* values that are true in both equations. **Figure 2-1** illustrates the solutions for two equations: $x + y = 6$ and $-3x + y = 2$. Where the lines cross is the solution for the simultaneous equations.

Example:

Start with the equations shown on the graph:

$$x + y = 6 \quad \text{and} \quad -3x + y = 2$$

$x + y = 6$ can also be written $y = 6 - x$

Substitute $6 - x$ into the second equation every time y appears. The second equation becomes:

$$-3x + 6 - x = 2$$

Combine like terms and solve for x :

$$-4x = -4, \quad x = 1$$

Knowing that $x = 1$, x can now be substituted into either equation to find the value of y and thus, the solution.

$$x + y = 6 \quad 1 + y = 6 \quad y = 5$$

The solution for the simultaneous equations:

$$x + y = 6 \quad \text{and} \quad -3x + y = 2 \quad \text{is:} \quad x = 1 \quad \text{and} \quad y = 5$$

A solution to simultaneous equations by elimination is similar. Examine the two equations for variables that will be eliminated if a mathematic function is applied. In this case, if the second equation is subtracted from the first equation, y will be eliminated leaving only x . Performing the arithmetic results in $x = 1$. Then, using either equation, 1 can be substituted wherever x occurs to find the value of y (5). Therefore, the solution is: $x = 1$ and $y = 5$.

Sometimes, simultaneous equations have no solution such as when the solution lines plotted on a graph result in parallel lines. Therefore, there will be no value for the variables that solves both equations.

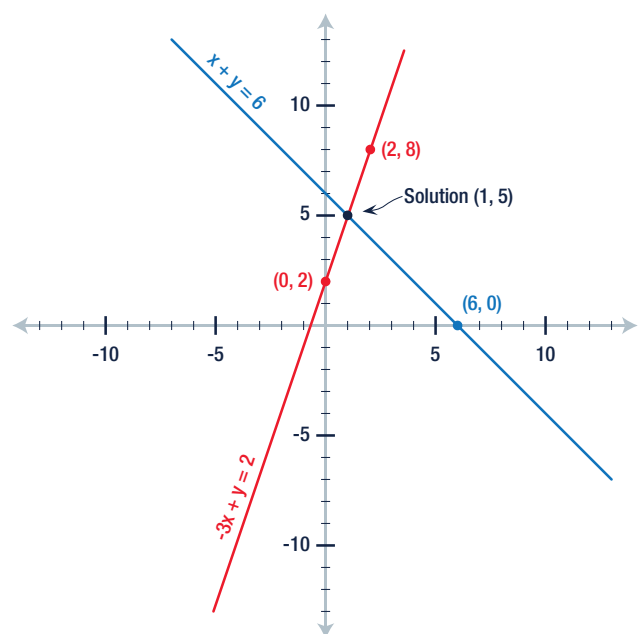


Figure 2-1. A solution for two simultaneous equations.

MORE ALGEBRAIC FRACTIONS

We saw earlier how to deal with brackets, factorizing algebraic expressions and solving equations. We can now look at some examples of adding, subtracting, multiplying and dividing slightly more complicated algebraic fractions. To add/subtract we use a common denominator as before.

Example:

Simplify the expression $\frac{a}{b} - \frac{a^2}{b^2}$

A common denominator is b^2 , so this fraction becomes

$$\frac{ab - a^2}{b^2}$$

Example:

Simplify the expression $\frac{1}{x + \frac{1}{1-x}} - 1$

At first glance this question may look difficult but it becomes easier to simplify if it is broken down into smaller parts.

Firstly, recall the definition of a reciprocal.

The reciprocal of $\frac{1}{1-x}$ is $1-x$.

The expression now becomes: $\frac{1}{x + 1-x} - 1$

which equals $\frac{1}{1} - 1$

which has the value zero.

Example:

Solve for x : $\frac{3}{x} + \frac{1}{x-2} = 2$

We could add the fractions on the left hand side or multiply all terms by $x(x-2)$ this will remove the fractions.

Multiplying all terms by $x(x-2)$ gives:

$$\frac{3}{x} x(x-2) + \frac{1}{(x-2)} x(x-2) = 2x(x-2)$$

Notice the extra brackets in the fraction $\frac{1}{(x-2)}$, they make it clear that all of $(x-2)$ divides in to 1.

Now divide as shown below:

$$\frac{3}{x} x(x-2) + \frac{1}{(x-2)} x(x-2) = 2x(x-2)$$

Then:

$$\begin{aligned} 3(x-2) + 1x &= 2x(x-2) \\ 3x - 6 + x &= 2x^2 - 4x \\ 4x - 6 &= 2x^2 - 4x \end{aligned}$$

Subtract $4x$ from each and add 6 to each side to get:

$$2x^2 - 8x + 6 = 0$$

Divide both sides by 2:

$$x^2 - 4x + 3 = 0$$

Factorize:

$$(x-3)(x-1) = 0$$

To get two solutions:

$$x = 3 \quad \text{and} \quad x = 1$$

INDICES AND POWERS IN ALGEBRA

In algebra, indices or powers are manipulated in the same manner as described in *sub-module 01 Arithmetic*. Refer to that section for any numerical explanation needed. The rules of Indices are given in **Figure 2-2**.

We will now look at lots of examples using these rules of indices, starting with some simple ones. An explanation of how the rules are being used is included. It is very important to be able to use these rules, not just for simplifying expressions or solving equations but also for rearranging equations.

RULES OF INDICES

1.	$a^p a^q = a^{p+q}$
2.	$\frac{a^q}{a^p} = a^{p-q}$
3.	$(a^p)^q = a^{pq}$
4.	$a^0 = 1$
5.	$a^{-p} = \frac{1}{a^p}$
6.	$a^{\frac{1}{p}} = \sqrt[p]{a}$
7.	$a^{\frac{p}{q}} = \sqrt[q]{a^p}$
8.	$(ab)^p = a^p b^p$
9.	$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

Figure 2-2. Rules of Indices table.

We will see how to rearrange equations after the section on Logarithms.

Example:
Simplify $a^2 a^5$

The base, a , is multiplied by itself, a , the number of times indicated by the exponent:

$$a^2 = a \times a \quad a^3 = a \times a \times a \quad a^4 = a \times a \times a \times a$$

So $a^2 a^5 = (aa) (aaaaa) = a^7$

Example:
Simplify $a^3 a^{-6}$

There are two of ways we can look at this question:

$$\begin{aligned} & a^3 a^{-6} \\ &= a^{3-6} \\ &= a^{-3} \text{ using rule 1} \end{aligned}$$

Or:

$$\begin{aligned} &= \frac{a^3}{a^6} \text{ using rule 5} \\ &= \frac{aaa}{aaaaaa} = \frac{1}{a^3} \text{ using definition of exponents} \\ &\quad \text{and dividing} \\ &= a^{-3} \end{aligned}$$

Example:
Simplify $\frac{x^2 y^6}{x y^5}$

Note that any number to the power of 1 is itself.

$$\begin{aligned} \frac{x^2 y^6}{x y^5} &= x^{2-1} y^{6-5} \text{ using rule 2} \\ &= x^1 y^1 \\ &= xy \end{aligned}$$

Example:
Simplify $\frac{a^2 2^2 b^5}{ab^7}$

$$\frac{a^2 2^2 b^5}{ab^7} = a^{2-1} 4b^{5-7} = 4a^1 b^{-2} = \frac{4a}{b^2} \text{ using rule 2 and 5}$$

Example:
Simplify $(x^3 y^{-2})^4$

$$\begin{aligned} (x^3 y^{-2})^4 &= x^{3 \times 4} y^{-2 \times 4} \text{ using rule 8} \\ &= x^{12} y^{-8} \text{ using rule 3} \\ &= x^{12} y^{-8} \end{aligned}$$

Example:
Simplify as far as possible $\frac{x^2 y^5 + x^4 y}{x^4 y}$

This example is different from the others in that it has a + sign in the numerator. This means that the denominator divides both terms in the numerator.

$$\begin{aligned} \frac{x^2 y^5 + x^4 y}{x^4 y} &= \frac{x^2 y^5}{x^4 y} + \frac{x^4 y}{x^4 y} \\ &= x^{2-4} y^{5-1} + x^{4-4} y^{1-1} \\ &= x^{-2} y^4 + x^0 y^0 \\ &= x^{-2} y^4 + 1 \text{ using rule 4} \end{aligned}$$

Example:
Simplify $\frac{9(3x)^0 xy}{(3x)^2}$

$$\begin{aligned} \frac{9(3x)^0 xy}{(3x)^2} &= \frac{9 \times 1 \times xy}{9x^2} \text{ using rule 4} \\ \frac{9 \times 1 \times xy}{9x^2} &= \frac{xy}{x^2} \text{ divide above and below by 9} \\ \frac{xy}{x^2} &= x^{1-2} y = x^{-1} y \text{ using rule 1} \end{aligned}$$

Example:

Simplify $\sqrt[3]{x^2y^3x^4y^{-6}}$

Using rule 7 we can write

$$\sqrt[3]{x^2} = x^{\frac{2}{3}}$$

$$\sqrt[3]{x^2y^3x^4y^{-6}} = x^{\frac{2}{3}}y^3x^{\frac{4}{3}}y^{-6} = x^{\frac{4}{3}}y^{-3}$$

It is well worth explicitly pointing out the following, since they are used so often in algebra.

$$(\sqrt{x})^2 = x$$

$$(\sqrt[3]{x})^3 = x$$

USING RULES OF EXPONENTS TO SOLVE EQUATIONS

When two numbers in exponent form are equal i.e. when $x^a = y^b$ then it must be true that $x = y$ and $a = b$.

We use this to solve some equations.

Example:

Solve for $4^x = \frac{1}{8}$ for x .

We first try to find a common base. We note that $4 = 2^2$ and $8 = 2^3$, so 2 is the common base.

Therefore:

$$(2^2)^x = \frac{1}{2^3}$$

$$2^{2x} = 2^{-3}$$

Since the bases are equal we can now equate the powers.

$$2x = -3 \text{ and } x = \frac{-3}{2} = -1.5$$

LOGARITHMS

An index or exponent tells the user how many times to multiply a number (the base) by itself. A *logarithm* answers the question: how many of one number are multiplied to arrive at another number? For example: how many 4's are multiplied to get 16? The answer, 2, is the logarithm. This is written as follows:

$$\log_4(16) = 2$$

We read this as "log to the base 4 of 16 equals 2".

Logarithms (commonly known as Logs) and exponents are related. Written algebraically, $4^2 = 16$. 2 is the logarithm. In general, it can be said that:

$$\text{Given that } b^x = y \text{ then } \log_b y = x$$

Examples:

$$10^2 = 100 \text{ can be written } \log_{10} 100 = 2$$

$$8^2 = 64 \text{ can be written } \log_8 64 = 2$$

$$3^4 = 81 \text{ can be written } \log_3 81 = 4$$

$$2^5 = 32 \text{ can be written } \log_2 32 = 5$$

When a logarithm is written without a base, it is implied that the base is 10.

$$\log(100) = 2 \text{ is actually } \log_{10}(100) = 2$$

This means that the number of times 10 needs to be multiplied by itself to equal 100 is 2 times.

There are a special group of logs called Natural Logarithms. Their base is a number that is called 'e'. 'e' is a constant and it has the value 2.718 28 to five decimal places. You do not need to remember the value of 'e'. Natural logs then, have 'e' as their base, so they could be written \log_e , however they are so widely used and so important that they are given the symbol \ln . (That is a lower case L followed by an n).

So, if $\ln x = y$ this implies that $e^y = x$

A benefit of using logarithms is that complicated calculations can be simplified, especially problems involving multiplication and division of extremely large or small numbers. For example, instead of multiplying or dividing large numbers, the logarithms of the numbers can be added when multiplying or subtracted when dividing. This results in much simpler calculations.

The rules of logs allow us to use logs in the way described above. The rules are shown in **Figure 2-3**.

LOGS TO BASE 10	NATURAL LOGS
$\log(MN) = \log M + \log N$	$\ln(MN) = \ln M + \ln N$
$\log\left(\frac{M}{N}\right) = \log M - \log N$	$\ln\left(\frac{M}{N}\right) = \ln M - \ln N$
$\log M^x = x \log M$	$\ln M^x = x \ln M$

Figure 2-3. The rules of logs table.

We can use logs to solve equations and to rewrite equations in a different form (*see end of this sub-module*).

Example:

Find the value x of given $\log_9 3 = x$.

First, we write the question in exponent form:

$$9^x = 3$$

we then note that $3^2 = 9$, so that:

$$(3^2)^x = 3^1$$

$$2x = 1$$

$$\text{and } x = \frac{1}{2}.$$

$$\text{Of course this is true, } \sqrt{9} = 9^{\frac{1}{2}} = 3$$

Example:

Solve the equation $\log x^2 + \log x = 2$

for x , giving your answer in the form 10^n , where n is a real number.

$$\log x^2 + \log x = \log(x^2 x) \text{ using Rule 1}$$

$$\log(x^2 x) = \log(x^3)$$

Putting this all together:

$$\log x^2 + \log x = \log(x^3)$$

$$\log(x^3) = 2$$

Writing in exponent form gives:

$$10^2 = x^3$$

Raising each side of the equation to the power of $\frac{1}{3}$ gives:

$$10^{\frac{2}{3}} = x$$

Example:

Write $4 \ln x = y$ in exponent form.

Solution:

$$4 \ln x = \ln x^4 \text{ Rule 3}$$

So that:

$$\ln x^4 = y$$

And:

$$e^y = x^4$$

Logarithms are especially useful when there is a need to compare numbers that have a very wide range between the smallest number and the largest number. Rather than use the actual values, it is helpful to compare the logarithms or to build a scale of comparison based on logarithms of the actual values.

The *decibel (dB)* scale for determining the sound level is one such means of comparison used in aerospace engineering. Logarithms are also used in navigational and aerodynamic calculations. When logs are used in this context however, either tables of log values or an electronic calculator would be necessary.

You will need to be able to:

- Change between writing a number in log form and exponential form.
- Solve equations using logarithms that do not require tables or an electronic calculator.
- Rewrite equations using the rules of logarithms.

TRANSPOSITION OF FORMULAE

The formula for converting temperature in degrees Celsius (C) to degrees Fahrenheit (F) is:

$$F = 1.8C + 32$$

For example, this allows us to convert 15° Celsius to 59° Fahrenheit. But what if we wanted to convert from Fahrenheit to Celsius? Our present formula tells us immediately the value of F in terms of the value of C. We say that F is the subject of the formula. What we want to do is make C the subject of the formula, i.e. get C on its own on one side of the formula. We must manipulate the formula to do this. There is only one rule to follow.

Whatever we do to one side of the equation, we must also do to the other side.

So if we add a quantity to one side, we add exactly the same quantity to the other. If we find the square root of one side, we find the square root of the other etc.

Let's take the example above and write the equation in terms of C.

$$F = 1.8C + 32$$

Subtract 32 from each side:

$$F - 32 = 1.8C$$

Divide both sides by 1.8:

$$\frac{F - 32}{1.8} = C$$

Example:

Make b the subject of the formula $p = \sqrt{a + b}$

We square both sides:

$$p^2 = a + b$$

Subtract a from both sides:

$$p^2 - a = b$$

Example:

Make x the subject of the formula $v = \frac{k}{\sqrt{x}}$

Multiply both sides by \sqrt{x}

$$v\sqrt{x} = k$$

Divide both sides by v :

$$\sqrt{x} = \frac{k}{v}$$

Square both sides:

$$x = \frac{k^2}{v^2}$$

Example:

Make x the subject of the formula $y = 4 - x^2$

Add x^2 to both sides:

$$y + x^2 = 4$$

Subtract y from both sides:

$$x^2 = 4 - y$$

Get the square root of both sides:

$$x = \sqrt{4 - y}$$

Example:

Make a the subject of the formula $s = ut + \frac{1}{2}at^2$

Multiply each side by 2:

$$2s = 2ut + at^2$$

Subtract $2ut$ from both sides:

$$2s - 2ut = at^2$$

Divide both sides by t^2 :

$$\frac{2s - 2ut}{t^2} = a$$

Example:

Make t the subject of the equation $\frac{a}{b} = c - \frac{m}{t}$

There are many ways to tackle this type of question.
Let's look at two different ways.

Method 1

Since we want t to be the subject, we might like the term it is in to be positive, so we will add $\frac{m}{t}$ to both sides:

$$\frac{a}{b} + \frac{m}{t} = c$$

Next subtract $\frac{a}{b}$ from both sides:

$$\frac{m}{t} = c - \frac{a}{b}$$

Multiply both sides by t :

$$m = \left(c - \frac{a}{b}\right)t$$

And divide both sides by $\left(c - \frac{a}{b}\right)$ to get:

$$\frac{m}{\left(c - \frac{a}{b}\right)} = t$$

Method 2

Here we will start by multiplying all terms by t :

$$\frac{a}{b}t = ct - m$$

Add m to both sides and subtract $\frac{a}{b}t$ from both sides:

$$m = ct - \frac{a}{b}t$$

Factorize t from both terms on the right:

$$m = \left(c - \frac{a}{b}\right)t$$

Divide both sides by $\left(c - \frac{a}{b}\right)$

$$\frac{m}{\left(c - \frac{a}{b}\right)} = t$$

NUMBER BASES

In this section we will look at Binary i.e. base 2 numbers, Octal i.e. base 8 numbers and Hexadecimal i.e., base 16 numbers.

THE BINARY NUMBER SYSTEM

The *binary number system* has only two digits: 0 and 1. The prefix in the word "binary" is a Latin root for the word "two" and its use was first published in the late 1700s. The use of the binary number system is based on the fact that switches or valves have two states: Open or Closed (ON or OFF). Primary uses of the binary number system include computer architecture and digital electronics. In computers, information is stored as a series of 0's and 1's, forming strings of binary numbers. Similarly, the binary number system is used in digital electronics because the two basic conditions of electricity, ON and OFF, can represent the two digits of the binary number system. When a switch is ON, it

represents the digit 1, and when it is OFF, it represents the digit 0. Millions and even billion of tiny switches are arranged so that digital electronic devices can perform the functions they do with a binary number system code.

Binary numbers are written with a subscript 2, for example 10011_2 and 1000110_2 are binary numbers.

When reading or pronouncing a binary number, it is common to simply say "1" or "0" moving from left to right until all the digits are pronounced.

To read 1011_2 , say, "one zero one one"

PLACE VALUES

The decimal number system used in every day life is a base-10 system. There are 10 digits available for use as place value holders; 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

DECIMAL PLACE VALUE CHART

10^7 = 10 000 000	10^6 = 1 000 000	10^5 = 100 000	10^4 = 10 000	10^3 = 1 000	10^2 = 100	10^1 = 10	10^0 = 1
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Figure 2-4 Place values of the decimal number system.

BINARY PLACE VALUE CHART

2^7 = 128	2^6 = 64	2^5 = 32	2^4 = 16	2^3 = 8	2^2 = 4	2^1 = 2	2^0 = 1
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Figure 2-5. Derivation of the place values of the binary number system.

When positioned in a number, they are also positioned to represent a place value. If 9 is exceeded, the place value resets to 0 and a 1 must be placed in the next place value column to the left. **Figure 2-4** illustrates the decimal number system place values. They are derived by sequentially raising 10 to a higher power moving from right to left. Thus, each position has a value 10 times that of the position to its right.

The binary number system is a base-2 system. There are 2 digits available for use as place value holders; 0 and 1. Each place value in the binary number system represents 2 raised to a sequentially higher power (from right to left). This is similar to the decimal system we use in everyday life. **Figure 2-5** illustrates the place values of the binary number system. It shows to what power 2 is raised to establish value and the decimal number system equivalent of each place. Each place value position has a value 2 times that of the position to its right. Remember, when writing binary numbers and placing digits in positions of place value, the only digits available are 0 and 1. To exceed 1, the place value is reset to 0 and a 1 is placed in the next place value column to the left. Place values are used to convert our every day decimal numbers to binary numbers.

Figure 2-6 illustrates how the decimal number in the left column is built from right to left by placing a 1 or a zero in the binary place value positions.

Here is a list of the first 20 binary numbers.

0, 1, 10, 11, 100, 101, 110, 111, 1 000, 1 001, 1 010, 1 011, 1 100, 1 101, 1 110, 1 111.

DECIMAL NUMBER	2^7 = 128	2^6 = 64	2^5 = 32	2^4 = 16	2^3 = 8	2^2 = 4	2^1 = 2	2^0 = 1
1								1
2							1	0
3							1	1
5						1	0	1
8					1	0	0	0
20				1	0	1	0	0
35			1	0	0	0	1	1
96		1	1	0	0	0	0	0
100		1	1	0	0	1	0	0
200	1	1	0	0	1	0	0	0
255	1	1	1	1	1	1	1	1

Figure 2-6. Use of binary number system place values to write various decimal numbers in binary (base₂).

THE OCTAL NUMBER SYSTEM

The octal number system has 8 digits 0, 1, 2..., 7.

Octal numbers are written using a subscript 8, for example 12 305₈. Each digit in an octal number has a place value that is a power of 8. For example, the octal number 457₈ has place values shown in the table below:

Place Value	$8^3 = 512$	$8^2 = 64$	$8^1 = 8$	$8^0 = 1$
Digit	4	5	7	2

One use of octal numbers is that they can be made from binary numbers by grouping consecutive binary digits into groups of three so the Octal number system provides an abbreviated method of writing Binary numbers.

Here is a list of the first 20 octal numbers:

0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24.

THE HEXADECIMAL NUMBER SYSTEM

The hexadecimal number system has 16 digits. Since there are only 10 digits that make up all numbers, to make 16 digits the letters A, B, C, D, E and F are used.

The first 16 hexadecimal numbers are 0, 1, 2..., 9, A, B, C, D, E, F.

A is equivalent to 10 in decimal, B is equivalent to 11 and so on.

Hexadecimal numbers are written using a subscript 16, for example A2C58₁₆.

Each digit in a hexadecimal number has a place value that is a power of 16.

For example, the octal number A4F7₁₆ has place values shown in the next table:

Place Value	$16^3 = 4096$	$16^2 = 256$	$16^1 = 16$	$16^0 = 1$
Digit	A	4	F	7

Hexadecimal numbers are used to describe locations in computer memory.

NUMBER SYSTEM CONVERSION

BINARY, OCTAL AND HEXADECIMAL TO BASE 10 (DECIMAL)

This is quite simple, just multiply each digit in the binary, octal or hexadecimal number by its place value and add.

Example:

Convert 4 135₈ to base 10

Let's use a table:

Place Value	$8^3 = 512$	$8^2 = 64$	$8^1 = 8$	$8^0 = 1$
Digit	4	1	3	5
Multiply	4×512	1×64	3×8	5×1

Then add:

$$(4 \times 512) + (1 \times 64) + (3 \times 8) + (5 \times 1) = 2141$$

$$4135_8 = 2141$$

Example:

Convert 1 001₂ to base 10

Place Value	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$
Digit	1	0	0	1
Multiply	1×8	0×4	0×2	1×1

Now multiply and add to get:

$$(1 \times 8) + (0 \times 4) + (0 \times 2) + (1 \times 1) = 8 + 1 = 9$$

$$\text{therefore } 1001_2 = 9$$

Example:

Convert A6D₁₆ to decimal:

Place Value	$16^2 = 256$	$16^1 = 16$	$16^0 = 1$
Digit	A	6	D
Multiply	10×256	6×16	13×1

Now multiply and add to get:

$$(10 \times 256) + (6 \times 16) + (13 \times 1) = 2669$$

$$\text{therefore } A6D_{16} = 2669$$

DECIMAL TO EITHER BINARY, OCTAL OR HEXADECIMAL

To convert a number from decimal to binary, continuously divide the number by 2, taking out the remainder at each step. Then read the remainders upwards.

To convert from decimal to octal use the same procedure but divide continuously by 8.

To convert from decimal to hexadecimal use the same procedure but divide continuously by 16.

The following examples will show this method.

Example:

Convert 10 to base 2.

We can divide this into steps.

Step 1:

$$\begin{array}{r} 2 \overline{)10} \\ \underline{5} \end{array} \leftarrow \text{Remainder 0}$$

Two divides into 10 five times with no remainder

Step 2:

$$\begin{array}{r} 2 \overline{)5} \\ \underline{2} \end{array} \leftarrow \text{Remainder 1}$$

Two divides into five twice with one remainder

Step 3:

$$\begin{array}{r} 2 \overline{)2} \\ \underline{1} \end{array} \leftarrow \text{Remainder 0}$$

Two divides into two once with no remainder

Step 4:

$$\begin{array}{r} 2 \overline{)1} \\ \underline{0} \end{array} \leftarrow \text{Remainder 1}$$

Two divides into one zero times with no remainder.

It is easier to write this all together as shown below.

$$\begin{array}{r} 2 \overline{)10} \\ 2 \overline{)5} \leftarrow \text{Remainder 0} \\ 2 \overline{)2} \leftarrow \text{Remainder 1} \\ 2 \overline{)1} \leftarrow \text{Remainder 0} \\ \underline{0} \leftarrow \text{Remainder 1} \end{array} \quad \begin{array}{c} \uparrow \\ \text{Read} \\ \text{answer} \\ \text{upwards} \end{array}$$

Therefore, $10 = 1010_2$

Example:

Convert 748 to base 8.

$$\begin{array}{r} 8 \overline{)748} \\ 8 \overline{)93} \leftarrow \text{Remainder 4} \\ 8 \overline{)11} \leftarrow \text{Remainder 5} \\ 8 \overline{)1} \leftarrow \text{Remainder 0} \\ \underline{0} \leftarrow \text{Remainder 1} \end{array} \quad \begin{array}{c} \uparrow \\ \text{Read} \\ \text{answer} \\ \text{upwards} \end{array}$$

Therefore, $748 = 1054_8$

Example:

Convert 256 to base 16.

$$\begin{array}{r} 16 \overline{)256} \\ 16 \overline{)16} \leftarrow \text{Remainder 3} \\ 16 \overline{)1} \leftarrow \text{Remainder 0} \\ \underline{0} \leftarrow \text{Remainder 1} \end{array} \quad \begin{array}{c} \uparrow \\ \text{Read} \\ \text{answer} \\ \text{upwards} \end{array}$$

Therefore, $256 = 103_{16}$

Example:

Convert 172 to base 16.

$$\begin{array}{r} 16 \overline{)172} \\ 16 \overline{)10} \leftarrow \text{Remainder 12 i.e. C} \\ 16 \overline{)0} \leftarrow \text{Remainder 10 i.e. A} \end{array} \quad \begin{array}{c} \uparrow \\ \text{Read} \\ \text{answer} \\ \text{upwards} \end{array}$$

Therefore, $172 = AC_{16}$

Computer and digital systems engineers may use other number systems to make their work easier. A binary number is typically much longer than its decimal equivalent. However, modern circuits have very fast switching speeds so that the length of binary numbers can be tolerated. This is especially true because of the reliability that is gained from a system that is built from components that are either 1 or 0, ON or OFF, that is, that either have voltage or do not have voltage.

Digital systems based on the binary number system will be studied in *Module 05* of this *Aviation Maintenance Technician Certification Series*.

Question: 2-1

Given the equation $x^2 + 3x - 10 = 0$, two solutions are:

- A. $x = 2, x = -10$
- B. $x = -2, x = 5$
- C. $x = 2, x = -5$

Question: 2-5

Two equations that share a variable and a common solution are known as:

- A. simultaneous.
- B. quadratic.
- C. linear.

Question: 2-2

The formula $T = 2\pi\sqrt{\frac{l}{g}}$ transposed for l is:

- A. $\frac{gT^2}{4\pi}$
- B. $\frac{gT}{2\pi}$
- C. $\frac{gT^2}{4\pi^2}$

Question: 2-6

Simplify the expression: $\frac{a^4b^{-4}c^3}{a^3b^{-2}c^{-5}}$

- A. $a^7b^{-6}c^8$
- B. $ab^{-2}c^8$
- C. $a^{-1}b^{-2}c^{-2}$

Question: 2-3

The binary number system is based on the fact that valves may be partially open or closed.

- A. True
- B. False
- C. Cannot be determined

Question: 2-7

Logarithms are often used to:

- A. calculate the angles in a triangle.
- B. simplify calculations involving extremely large or small numbers.
- C. convert numbers to scientific notation.

Question: 2-4

Solve the equation: $\frac{2}{x} \times \frac{3}{y} =$

- A. $\frac{2y}{3x}$
- B. $6xy$
- C. $\frac{6}{xy}$

Question: 2-8

Which of the following is another way of expressing $a^{\frac{1}{3}}$?

- A. $\sqrt[3]{a}$
- B. $3\sqrt{a}$
- C. $\sqrt{\frac{a}{3}}$

ANSWERS

Answer: 2-1

Answer is C.

Factorize using brackets to get:

$$(x - 2)(x + 5) = 0$$

then:

$$(x - 2) = 0 \text{ to give the solution } x = 2$$

$$(x + 5) = 0 \text{ to give the solution } x = -5$$

Answer: 2-2

Answer is C.

Given the formula: $T = 2\pi\sqrt{\frac{l}{g}}$

Divide both sides by: $2\pi\sqrt{\frac{l}{g}}$

Square both sides: $\frac{T^2}{2^2\pi^2} = \frac{l}{g}$

Multiply both sides by g : $\frac{gT^2}{4\pi^2} = l$

Answer: 2-3

Answer is A.

The binary numbering system is based on only two inputs using switches or valves to indicate open or closed, or in digital electronics indicating on or off.

Answer: 2-4

Answer is C.

Manipulating fractions in Algebra is the same as manipulating numbers. To multiply two fractions, first, multiply the numerators together: $2 \times 3 = 6$. This becomes the numerator. Then, multiply the denominators: $x \times y = xy$. This becomes the denominator.

So the answer is: $\frac{6}{xy}$.

Answer: 2-5

Answer is A.

For equations to work together, they must have at least one common variable. This makes them simultaneous. The two or more equations may also be known as a system of equations.

Answer: 2-6

Answer is B.

Given the expression: $\frac{a^4b^{-4}c^3}{a^3b^{-2}c^{-5}}$

and using rules of indices, the expression becomes:

$$\begin{aligned} & a^{4-3}b^{-4-(-2)}c^{3-(-5)} \\ & = a^1b^{-4+2}c^{3+5} \\ & = ab^{-2}c^8 \end{aligned}$$

Answer: 2-7

Answer is B.

Doing calculation with extremely large or extremely small numbers can be cumbersome and prone to errors due to the many decimal place value holders. Logarithms allow the numbers to be expressed in terms of their positive or negative exponents thus simplifying their manipulation.

Answer: 2-8

Answer is A.

When an exponent is a fraction, it is the same as if looking for the root of the number. Therefore, a radical sign could be used with the denominator of the fractional index describing what root is sought.

Question: 2-9

Which of the following is a linear equation?

- A. $x^2 = 2x - 2$
- B. $3y - 2x + 5 = 0$
- C. $x(x + 1) = 5$

Question: 2-11

Find the value of 10 raised to the negative sixth power.

- A. 0.000 010
- B. 0.000 001
- C. 0.000 1

Question: 2-10

Which of the following are not an example of a binary number?

- A. 121 102
- B. .001
- C. 11 100 110

Question: 2-12

The operations inside parenthesis are always performed first in algebraic equations.

- A. True
- B. False
- C. Cannot be determined

ANSWERS

Answer: 2-9

Answer is B.

When graphed, the solutions to a linear equation make a straight line. Variables in a linear equation are only raised to the first power to make this possible. Once a variable is raised to the second power, such as in answers A and C, the shape of the line representing the solutions to the equation is curved.

Answer: 2-10

Answer is A and B.

Remember there are only two available digits in a binary number, "1" and "0". There are also no decimals used in the binary system.

Answer: 2-11

Answer is B.

Remember that negative powers mean that you divide the number by itself the same amount of times as is equal to the power, which would be $10 \div 10 \div 10 \div 10 \div 10 \div 10$. Another way to calculate this would be to use its reciprocal or $1/10^6$, which is 1 divided by 1 000 000 or 0.000 001.

Answer: 2-12

Answer is A.

True. Always perform the mathematical operations inside parenthesis first in algebraic equations.



PART-66 SYLLABUS LEVELS

CERTIFICATION CATEGORY →

B1 **B2**

Sub-Module 03

GEOMETRY

Knowledge Requirements

3.1 - Geometry

- (a) Simple geometrical constructions;
- (b) Graphical representation; nature and uses of graphs, graphs of equations/functions;
- (c) Simple trigonometry; trigonometrical relationships, rectangular and polar coordinates.

1

1

2

2

2

2

GEOMETRY

Level 1

A familiarization with the principal elements of the subject.

Objectives:

- (a) The applicant should be familiar with the basic elements of the subject.
- (b) The applicant should be able to give a simple description of the whole subject, using common words and examples.
- (c) The applicant should be able to use typical terms.

Level 2

A general knowledge of the theoretical and practical aspects of the subject and an ability to apply that knowledge.

Objectives:

- (a) The applicant should be able to understand the theoretical fundamentals of the subject.
- (b) The applicant should be able to give a general description of the subject using, as appropriate, typical examples.
- (c) The applicant should be able to use mathematical formula in conjunction with physical laws describing the subject.
- (d) The applicant should be able to read and understand sketches, drawings and schematics describing the subject.
- (e) The applicant should be able to apply his knowledge in a practical manner using detailed procedures.

GEOMETRY IN AVIATION MAINTENANCE

In this sub-module we will first look at some properties of different shapes e.g. angles, number of sides. We will then consider *functions* and their graphs. Aircraft maintenance data is often presented on a graph. Explanation and examples of how graphs are constructed, used and how to interpret the vital information

conveyed by graphs are given in this sub-module. Since the use of geometry and calculations are important when fabricating parts for structural repair, simple trigonometry and trigonometric relationships are also explored. Trigonometric functions are also important in acoustics, electricity and electronics.

SIMPLE GEOMETRIC CONSTRUCTIONS

ANGLES

In *sub-module 01*, in the section on Areas, **Figure 1-15** shows the different types of triangle. We will now look at different types of angles. (**Figure 3-1**)

Angles are measured either in degrees or radians. **Figure 3-2** shows the angle of one radian.

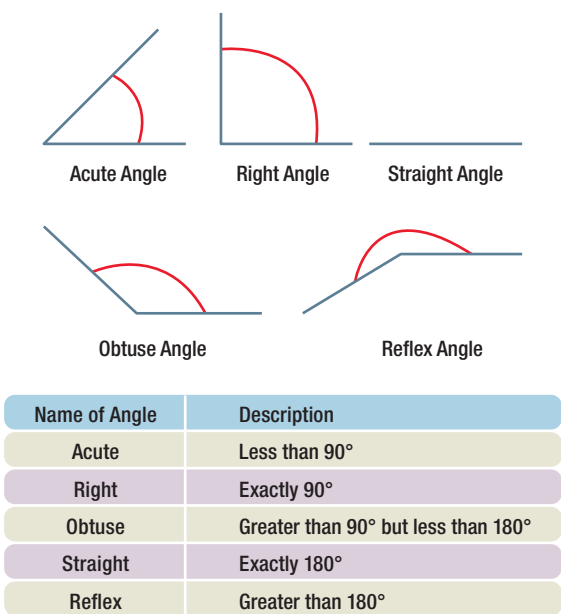


Figure 3-1. Various types of angles.

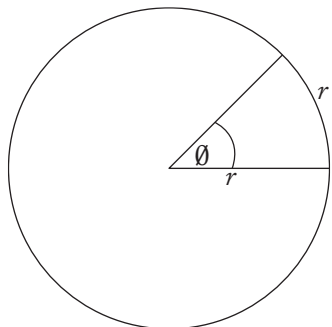


Figure 3-2. Showing an angle of one radian.

Degrees are given the symbol °. A degree is $\frac{1}{360}$ of a full circle. In other words a full circle is divided into 360°. The reason for this type of division is that the ancient Babylonians and Persians believed that there were 360 days in a year! Degrees are further divided into 60 minutes. Minutes have the symbol '. Each minute is divided into 60 seconds. Seconds are given the symbol ''.

An angle with the measure of 36 degrees, 16 minutes and 34 seconds is therefore written as 36°16' 34''. When using degree measure sometimes decimal points of degrees are used. For example:

$$72^{\circ}30' = 72.5^{\circ}$$

RADIANS

Radians have no symbol, sometimes however, to avoid confusion, they are written with the word 'radians' after the numerical value.

One radian is defined to be equivalent to the angle subtended at the center of a circle by an arc equal in length to the radius, r . There are 2π radians in a full circle. **Figure 3-3** shows a comparison of degree and radian measurement for every quarter circle.

Degrees	Radians
0°	0
90°	$\frac{\pi}{2}$
180°	π
270°	$\frac{3\pi}{2}$
360°	2π

Figure 3-3. Degree and radian measurement for every quarter circle.

CONVERTING BETWEEN DEGREES AND RADIANS

Degrees To Radians

To convert from degrees to radians (and vice versa) we use the fact that:

$$180^\circ = \pi$$

Dividing both sides by 180° gives:

$$1^\circ = \frac{\pi}{180}$$

So, to convert degrees to radians, we multiply by $\frac{\pi}{180}$.

Example:

Convert 27° and 195° to radians

$$27^\circ = 27 \times \frac{\pi}{180} \text{ radians} = \frac{3\pi}{20} \text{ radians}$$

$$195^\circ = 195 \times \frac{\pi}{180} \text{ radians} = \frac{13\pi}{12} \text{ radians}$$

Radians To Degrees

To convert from radians to degrees we also use the fact that:

$$\pi = 180^\circ$$

Dividing each side by π gives:

$$1 = \left(\frac{180}{\pi}\right)^\circ$$

So, to convert radians to degrees we multiply by $\frac{180}{\pi}$.

Example:

Convert 5 and $\frac{2\pi}{3}$ radians to degrees.

$$5 \text{ radians} = 5 \times \left(\frac{180}{\pi}\right)^\circ = \frac{900}{\pi}^\circ$$

$$\frac{2\pi}{3} \text{ radians} = \frac{2\pi}{3} \times \left(\frac{180}{\pi}\right)^\circ = \frac{360}{3} = 120^\circ$$

PROPERTIES OF SHAPES

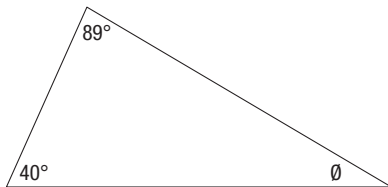
In *sub-module 01* we showed how to calculate the areas and volumes of different shapes. In this sections we will look at the properties of shapes e.g. number of sides, angles etc.

TRIANGLES

All triangles have three sides and the sum of the angles in any triangle is 180° . There are five types of triangle, as described in **Figure 3-4**. A diagram of these triangles is given in *sub-module 01*.

Example:

For the triangle shown below, what is the measure of the angle θ ?



Since the sum of the angles in a triangle is 180° ,

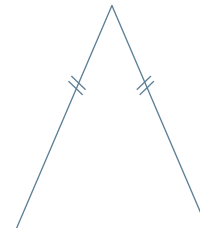
$$\theta = 180^\circ - (89^\circ + 40^\circ) = 51^\circ$$

Five Types of Triangles	
Right Angle	Has one 90° angle.
Equilateral	All angles 60° . All sides equal length.
Isosceles	Has two angles the same and two sides the same.
Scalene	Has all three angles and all three sides different.
Obtuse	Has one obtuse angle.

Figure 3-4. A description of the five types of triangles.

Example:

State the type of triangle shown below and give a reason for your answer.



This is an Isosceles triangle, it has two sides the same length.

FOUR SIDED FIGURES

There are six types of four sided shapes. (*Figure 3-5*)

SQUARE

The square has four sides of equal length. Each of the interior angles is 90° . The diagonals are equal and bisect each other. This means they divide each other equally.

RECTANGLE

The rectangle has two pairs of opposite and equal sides. Each of the interior angles is 90° . The diagonals are equal and bisect each other.

RHOMBUS

The rhombus has four sides of equal length. The diagonals are not equal in length. The diagonals bisect each other.

PARALLELOGRAM

The parallelogram has two pairs of opposite, equal and parallel sides. The diagonals are not equal in length. The diagonals bisect each other.

TRAPEZIUM

This shape has one pair of opposite sides that are parallel to each other.

KITE

A kite has two pairs of sides of equal length.

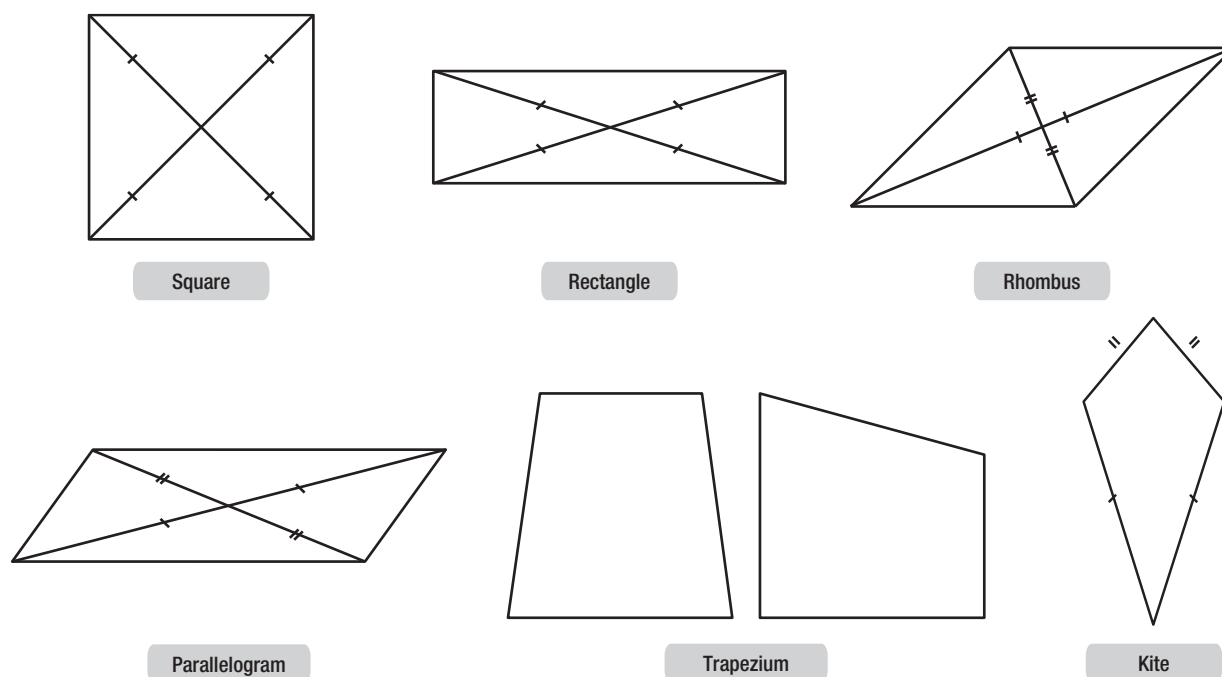


Figure 3-5. Six types of four sided shapes.

GRAPHICAL REPRESENTATIONS

Graphs and charts are pictorial presentations of data, equations and formulas. Through their use, the relationship between two or more quantities may be more clearly understood. *Graphs* show certain conditions or relationships at a glance that might require considerable time to ascertain from the same information in a written description or table.

INTERPRETING GRAPHS AND CHARTS

The words graph and chart are synonymous. Aviation maintenance technicians are more concerned with reading and interpreting the information presented on graphs and charts than they are in actually constructing them. The type of graph in *Figure 3-6* is commonly used. It presents two different but related variables to be considered by the user.

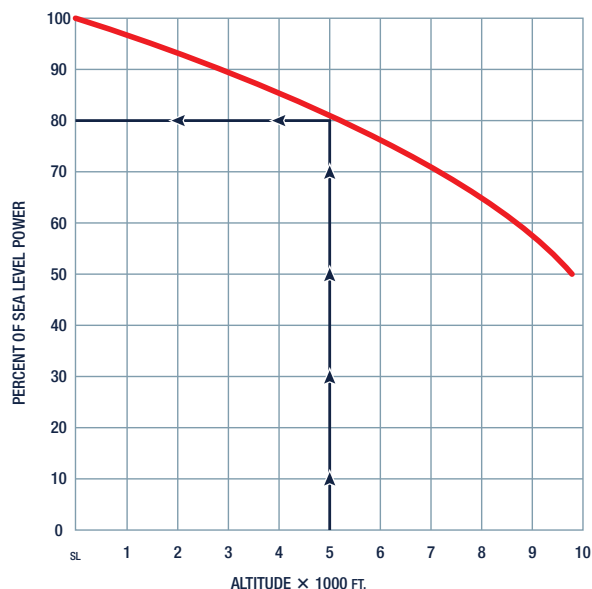


Figure 3-6. Engine horsepower versus altitude.

This particular graph is useful in analyzing the decrease in engine power as altitude increases.

One variable, altitude, is presented on the horizontal axis. It increases as one moves to the right. The other variable, the percent of sea level power, is presented on the vertical axis. It increases as one moves up the axis. The line is drawn on the graph in the precise position that shows the relationship between these two variables. When one of the variables is known, the other can be determined. This is done by moving vertically or horizontally from where the curved line and the known quantity intersect, to the axis of the unknown variable. Read the scale on the axis to obtain the unknown value.

An example comparison of altitude and percent of sea level power is indicated in *Figure 3-6* by the arrows. The chart is being used to find the available power when flying at 5 000 feet. At 5 000 feet altitude (read off of the scale on the horizontal axis), the user moves directly up until the 5 000 foot line intersects the curved line. From this point, the user moves directly left to the vertical axis where the percent of power value can be read off the vertical scale. The arrows show that 80 percent of sea level power is available.

There is virtually no limit to the number of two variable relationships that can be depicted with a graph or chart in this manner.

In *Figure 3-7*, Boeing Aircraft Company compares two variables that are used by the technician to properly service the main landing gear strut with nitrogen. Included in the diagram with the chart are the servicing instructions and procedures to guide the technician when interpreting the data on the chart. A drawing of where dimension "x" should be measured is also included. Dimension "x" is one of the variables on this two-variable chart. The other variable is the inflation pressure to which the strut should be serviced.

A unique feature on this chart is that there are two curved lines. They are to be used as the instructions indicate. Also, one of the lines is thick. This presents a range of intersecting values rather than a single point. However, the basic procedure for using the chart remains as presented in the previous example. Always follow the manufacturer's instructions when further instructions on using a graph or chart are presented.

GRAPHS WITH MORE THAN TWO VARIABLES

A graph or chart with more than two variables may be encountered. This type of graph is shown in *Figure 3-8* and is called a *nomogram*. When interpreting this type of graph, start with the known variables and move horizontal or vertically until all the known variables have been included in the movements. The value of the unknown variable should be able to be read off of one of the scales at the edge of the graph.

Follow the example below to learn how this is done. Also, always be sure to read and follow any manufacturer's instructions that accompany any graph or chart.

Example:

An aircraft reciprocating engine has a 1 830 cubic inch displacement and develops 1 250 brake-horsepower at 2 500 rpm.

What is the brake mean effective pressure?

Explanation:

Use the following steps:

1. Find the 1 250 brake-horsepower vertical line at the top of the chart, and follow this line down until it intersects the diagonal line for the 1 830 cubic inch displacement engine.

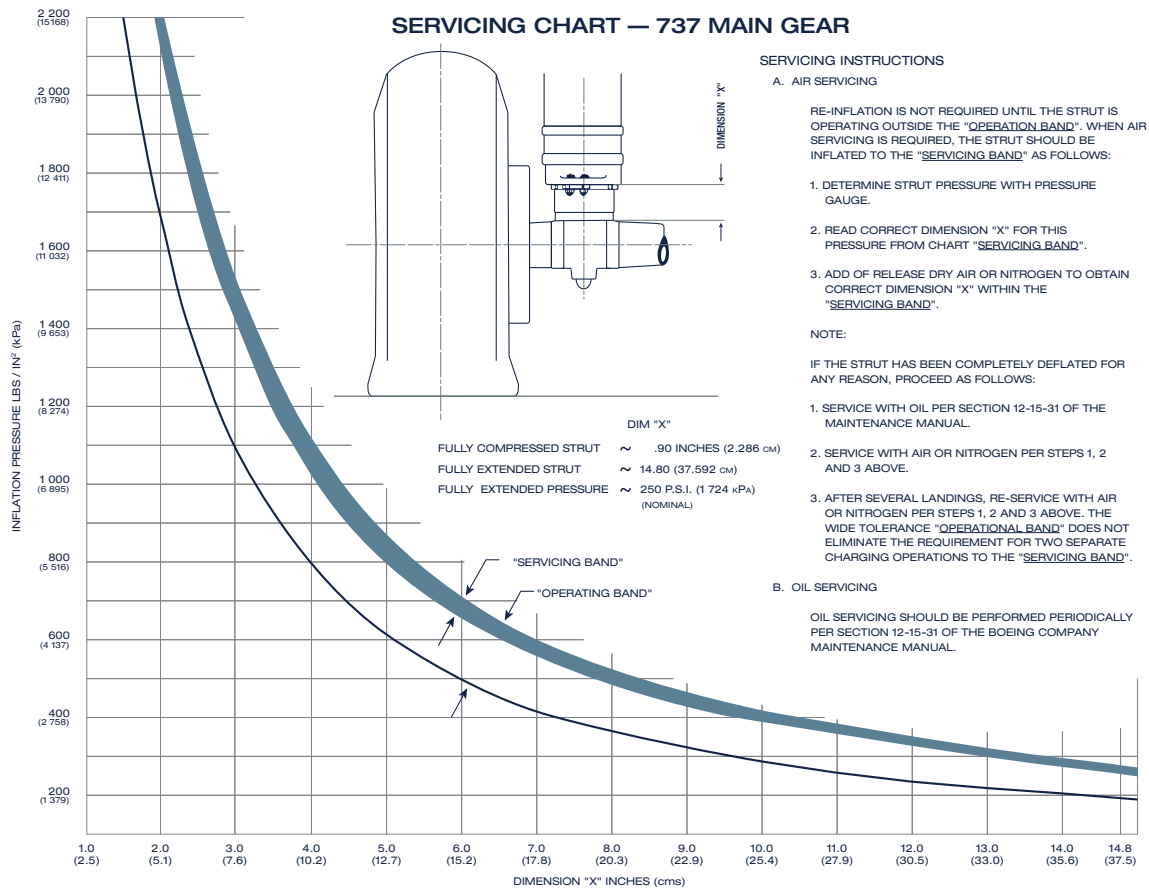


Figure 3-7. A Boeing Aircraft Company main landing gear servicing chart.

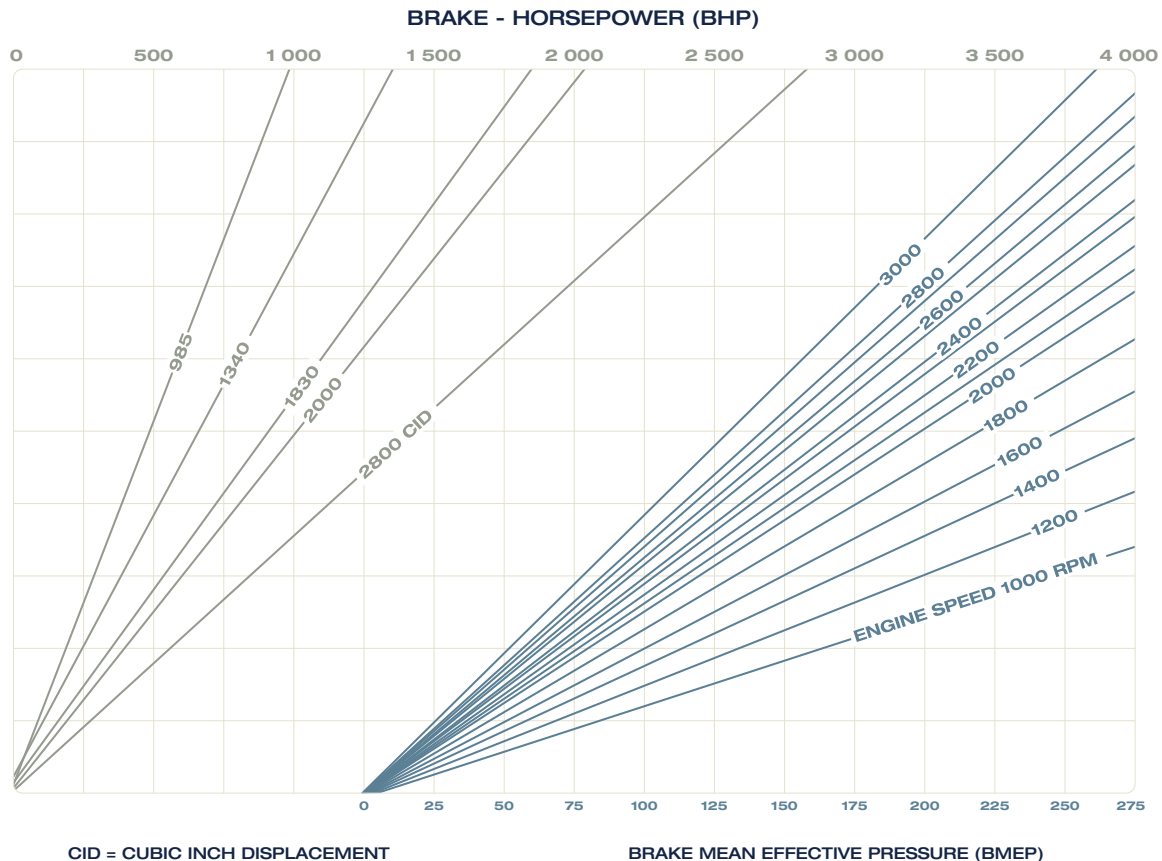


Figure 3-8. A brake horsepower nomogram for 5 engines with 3 variables on the same graph.

2. Draw a horizontal line from this intersection point to the right until it intersects the diagonal line for 2 500 rpm.
3. From the intersection of the horizontal line and the rpm diagonal line, draw a line vertically downward to the BMEP scale.
4. You will find that the vertical line touches the BMEP scale just about on the 217 psi BMEP line.

CARTESIAN COORDINATE SYSTEM

On any graph by extending the horizontal axis to the left and the vertical axis below the horizontal axis, the graph can encompass negative values. A *rectangular coordinate system*, known as the *Cartesian Coordinate System*, is used to define the plane, that is, to create the space for a graph. It consists of two intersecting axes at right angles to each other. The x axis is horizontal and the y axis is vertical. The point where the axes intersect is called the Origin. Each axis is calibrated in even increments, but the axes do not have to be marked in the same increments as each other.

To locate a point on the coordinate system, an ordered pair is required. This consists of two values presented as (x, y) . The first value is located horizontally along the x axis. The second value is located vertically along the y axis. In *Figure 3-9*, the ordered pair $(3, 5)$ has been plotted.

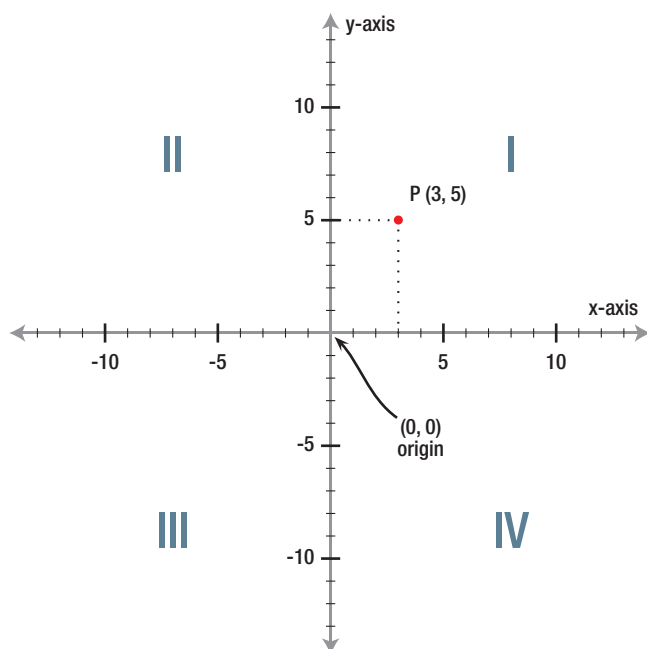


Figure 3-9. The rectangular or Cartesian Coordinate System for plotting data for graphs.

The large roman numerals in the diagram label the four quadrants of the graph. These labels are not normally shown. In the first quadrant, both coordinates are positive, in the second quadrant x -coordinates are negative and y -coordinates positive, in the third quadrant both coordinates are negative and in the fourth quadrant, x -coordinates are positive and y -coordinates negative. This is summarized in *Figure 3-10*. *Figure 3-11* illustrates the rectangular coordinate system.

QUADRANT	X = VALUES	Y = VALUES
I	> 0	> 0
II	< 0	> 0
III	< 0	< 0
IV	> 0	< 0

Figure 3-10. Positive and negative coordinate values in the quadrants of the rectangular or Cartesian Coordinate System.

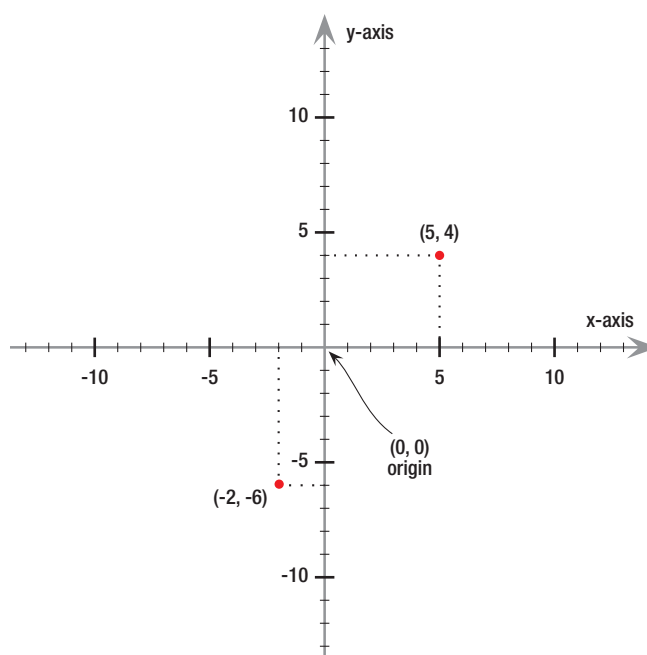


Figure 3-11. Rectangular or Cartesian Axes.

GRAPHS OF EQUATIONS AND FUNCTIONS

The graph of an equation is made up of the set of points where the equation is true. Rectangular axes are used. Solving a two variable equation by giving one of the variables a series of values results in ordered pairs that can be plotted. Then, a line or a curve can be drawn through these representative points to produce the graph of the equation.

WHAT IS A FUNCTION?

A function relates an input to an output. It can be said that a function defines the relationship between the input (usually x) and the output (usually y). In mathematics we say y is a function of x . What this means is that we have an equation that links x and y , and when we choose a value of x we can find the value of y .

For example, for the function $y = 2x + 2$, we can see in the table below the values that have been calculated for y when we have chosen values for x between -2 and $+2$.

x	-2	-1	0	1	2
$2x$	-4	-2	0	2	4
$+2$	$+2$	$+2$	$+2$	$+2$	$+2$
$y = 2x + 2$	-2	0	2	4	6

We say that y is a function of (x), and write this as:

$$y = f(x).$$

So it is just as correct to write $f(x)$ as to write y . We now have a set of pairs of x and y values. We have a set of ordered pairs. They are $(-2, -2)$, $(-1, 0)$, $(0, 2)$, $(1, 4)$ and $(2, 6)$.

Another example of a function is:

$$f(x) = x^2$$

This is read: "the function of x equals x squared" or " f of x equals x squared". x^2 is just an example of a squaring function. But the function could be just about any expression that when an input is received, an output is created. $f(x) = x^3 + 1$ is a function as is $f(x) = 1 - x + x^2$. The trigonometric ratios introduced in the next section are also functions. When an input to a function is made, it produces a single corresponding output. Together, the

input and the output create an ordered pair. By plotting numerous inputs and outputs (*ordered pairs created by the function*) a representative line or curve on a graph can be drawn. Two very common functions are shown in the table below.

Name		Shape	General Formula
Linear Function	Highest power of x is 1	Straight Line	$y = mx + c$
Quadratic Function	Highest power of x is 2	Parabola	$y = ax^2 + bx + c$

Note when drawing graphs we choose x and calculate y . If we want to choose values of x between -3 and $+3$ inclusive, we write this as $-3 \leq x \leq 3$. This is read as -3 is less than or equal to x is less than or equal to 3 .

LINEAR FUNCTIONS AND THEIR GRAPHS

Example:

Draw a graph of the function:

$$y = 4x + 2, \text{ for } -3 \leq x \leq 3$$

Create a table and plot the points on a graph.
(Figure 3-12)

x	-3	-2	-1	0	1	2	3
$4x$	-12	-8	-4	0	4	8	12
$+2$	2	2	2	2	2	2	2
y	-10	-6	-2	2	6	10	14

Example:

Graph $f(x) = x + 3$

Choose values for x and perform the function to create ordered pairs.

Input x	Function Output	Ordered Pairs
-3	0	$(-3, 0)$
-2	1	$(-2, 1)$
-1	2	$(-1, 2)$
0	3	$(0, 3)$
1	4	$(1, 4)$
2	5	$(2, 5)$
3	6	$(3, 6)$

Finally, plot the points on a graph (Figure 3-13).

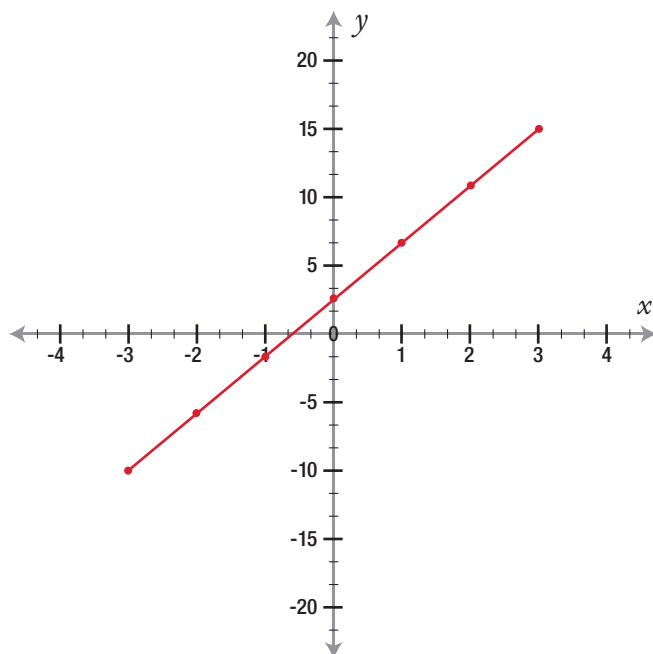


Figure 3-12. Graph of the function $y = 4x + 2$ for $-3 \leq x \leq 3$.

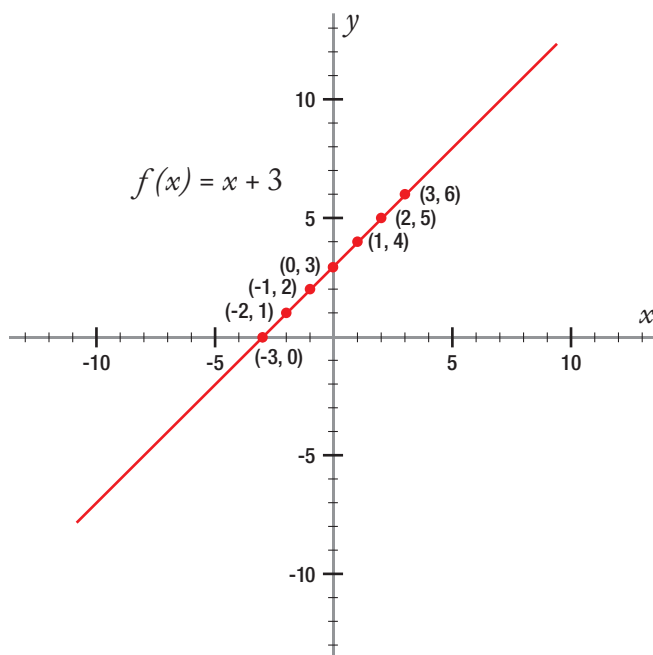


Figure 3-13. Graph of the function $f(x) = x + 3$.

SLOPE OF A LINE

When dealing with straight line graphs we are often interested in the slope or gradient of the graph. The slope gives us a value on how fast the graph is increasing or decreasing. It gives us a value for how fast y is changing with respect to x .

Example 1:

If a line has a slope or gradient of 4, this means that x is increasing at a rate 4 times faster than x .

Example 2:

If a line has a slope of 10, this means that y is increasing 10 times faster than x .

Example 3:

If a line has a gradient of -3, it means that y is decreasing three times faster than x is increasing.

Example 4:

If a line has a slope of zero it is parallel to the x axis.

In two examples above, the slopes are positive.

Example:

Plot a graph of the function:

$$y = -2x + 4$$

for

$$-3 \leq x \leq 3$$

We make a table of x and y values.

x	-3	-2	-1	0	1	2	3
$-2x$	6	4	2	0	-2	-4	-6
$+4$	4	4	4	4	4	4	4
y	10	8	6	4	2	0	-2

Then plot the graph. (Figure 3-14)

This line has a negative slope.

Example:

Plot a graph of the function:

$$y = -3$$

for

$$-3 \leq x \leq 3$$

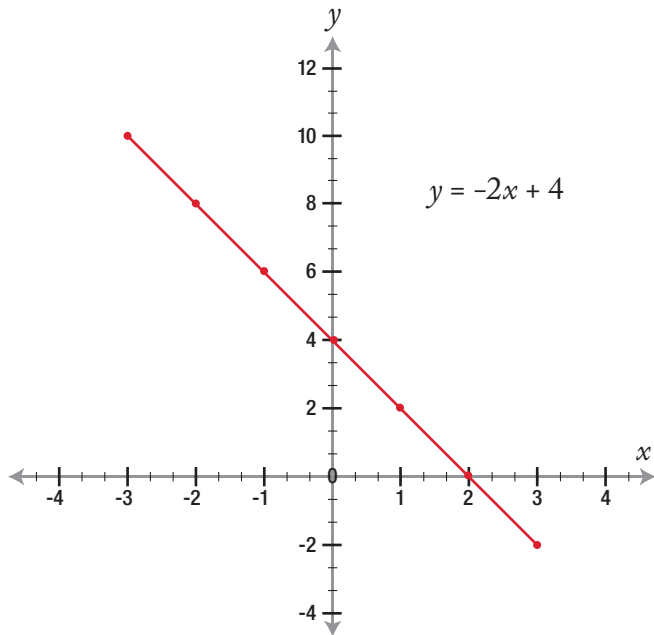


Figure 3-14. Graph of the function $y = -2x + 4$ for $-3 \leq x \leq 3$.

We make a table of x and y values.

x	-3	-2	-1	0	1	2	3
y	-3	-3	-3	-3	-3	-3	-3

Then plot the graph (Figure 3-15).

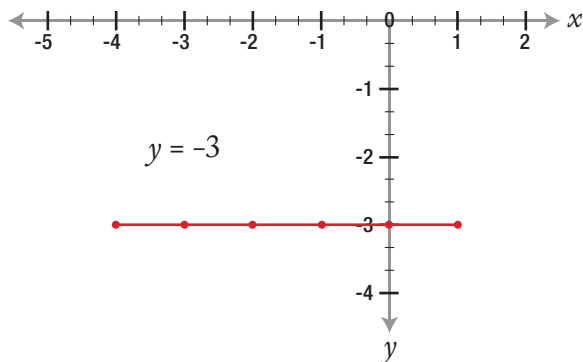


Figure 3-15. Graph of the function $y = -3$ for $-3 \leq x \leq 3$.

The graph of $y = -3$, is an example of a constant function. Constant means that it does not change, y is always -3 . The graph is parallel to the x -axis. This line has a slope of zero, just as level ground has a slope of zero.

We have mentioned that the general form of a linear function is $y = mx + c$. Now let's see what this means. Comparing the example $y = 4x + 2$ shown above, to $y = mx + c$, we can see that $m = 4$ and $c = 2$.

Here's a few more examples of linear functions showing their values of m and c .

	m	c
$y = 3x + 9$	3	9
$y = 2x - 1$	2	-1
$y = -4x + 1$	-4	1
$y = -6x - 5$	-6	-5

When the function of a line is written in the form:

$$y = mx + c \quad m \text{ is the slope of the line.}$$

Example:

What is the slope of the line given by the equation:

$$2y - 4 = 8x$$

We cannot read the slope from the equation given. We must rewrite it in terms of y .

Add 4 to both sides:

$$2y = 8x + 4$$

Divide both sides by 2:

$$y = 4x + 2$$

The slope is 4.

You need to know the following for examinations:

For any two lines with slopes m_1 and m_2 .

If $m_1 = m_2$ the lines are parallel.

If $m_1 = -\frac{1}{m_2}$ then the two lines are at right angles to each other i.e. if they are perpendicular.

Y-AXIS INTERCEPT

Where a graph crosses the y -axis is called the y -axis intercept. When a linear function is written in the form $y = mx + c$, the number c is where the line crosses the y -axis.

Example:

The graph of the function $y = -4x + 8$ is shown in Figure 3-16. Write out the coordinates where the graph crosses the x -axis and the y -axis.

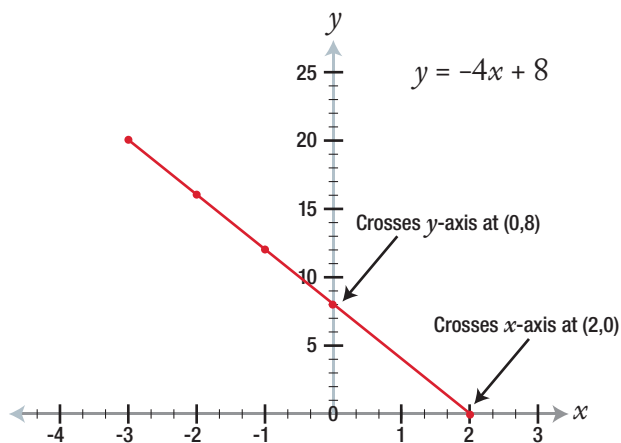


Figure 3-16. Graph of the function $y = -4x + 8$.

Where the graph crosses the x -axis, y has the value of zero. The point is $(2, 0)$. Where the graph crosses the y -axis, x has the value of zero. The point is $(0, 8)$.

Example:

Where does the graph of the function $y = -4x - 24$ cross the x -axis?

It crosses the x -axis when $y = 0$.

Therefore it crosses the x -axis when $0 = -4x - 24$

Solving for x gives $x = -6$.

QUADRATIC FUNCTIONS

The general form of a quadratic function is:

$$y = ax^2 + bx + c$$

We graph them in exactly the same way as linear functions.

Example:

Graph $f(x) = x^2$

Choose values for x and perform the function to create ordered pairs.

Input x	Function Output	Ordered Pairs
-3	9	$(-3, 9)$
-2	4	$(-2, 4)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	4	$(2, 4)$
3	9	$(3, 9)$

Now plot the ordered pairs created by the function on a graph as shown in **Figure 3-17**.

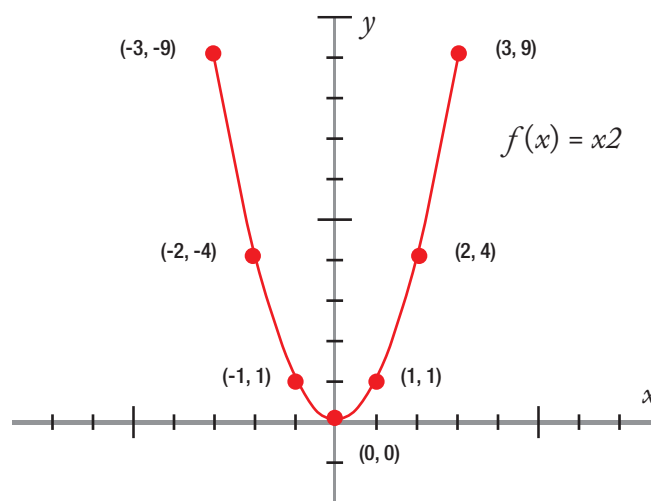


Figure 3-17. Graph of a function $f(x) = x^2$.

Example:

Graph the function $y = x^2 - x - 6$.

We make a table of values:

x	-3	-2	-1	0	1	2	3	4
y	6	0	-4	-6	-6	-4	0	6

Then draw the graph (**Figure 3-18**).

Note that the graph crosses the x -axis twice, namely when $x = -2$ and $x = 3$. In *sub-module 02* we looked at solving quadratic equations by factorizing.

We solved equations such as:

$$x^2 - x - 6 = 0$$

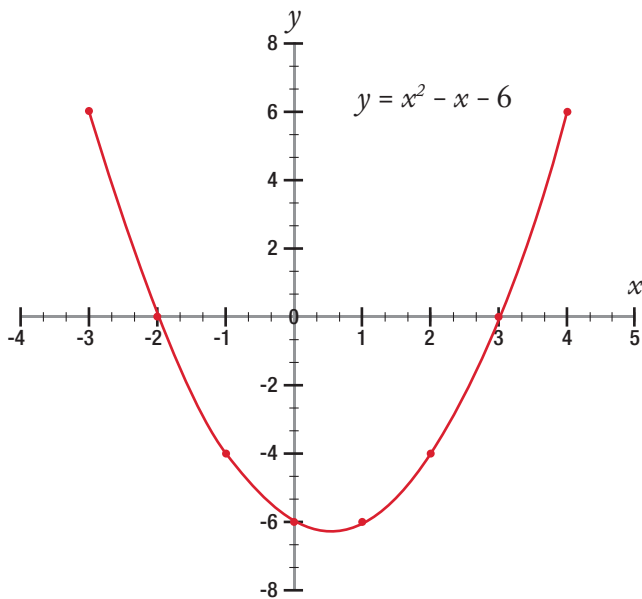


Figure 3-18. Graph of the function $y = x^2 - x - 6$.

One method we used was to draw brackets in the form:

$$(x + ?)(x + ?) = 0$$

If we factorize the equation above using this method we get:

$$(x - 3)(x + 2) = 0$$

It follows from this that:

$$(x - 3) = 0, \text{ giving } x = 3$$

$$(x + 2) = 0, \text{ giving } x = -2$$

These are exactly the same as the solutions we can read from the graph.

Important note, solutions to the equation $ax^2 + bx + c = 0$ are found where the graph of the function $y = ax^2 + bx + c$ crosses the x -axis.

Here is an example of a quadratic equation.

Example:

The graph of the function $y = f(x)$ is shown in **Figure 3-19**. For what values of x is $f(x) = 0$?

$f(x) = y = 0$ where the graph crosses the x -axis. In this case it crosses the x -axis at $x = -2$ and at $x = 4$.

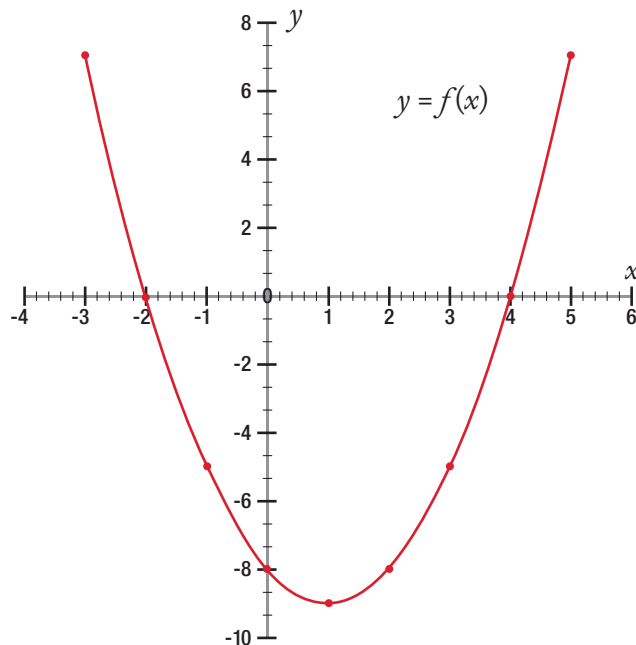


Figure 3-19. Graph of the function $y = f(x)$.

Let's look another example of a quadratic function.

Example: Draw a graph of the function:

$$y = x^2 + 3x + 4$$

We create a table of values:

x	-4	-3	-2	-1	0	1	2
y	8	4	2	2	4	8	14

Then draw a graph as shown in **Figure 3-20**. Note that this graph does not cross the x -axis. This means that there are no real solutions to the equation.

$$0 = x^2 + 3x + 4$$

We have drawn three graphs of quadratic equations:

1. This graph crossed the x -axis once only:

$$y = x^2$$

2. This graph crossed the x -axis twice:

$$y = x^2 - x - 6$$

3. This graph did not cross the x -axis.

$$y = x^2 + 3x + 4$$

All of the three graphs (**Figures 3-18 through Figure 3-20**) have a minimum point.

Without showing the table of values, here are the graphs of three more quadratic functions. In each of the three functions shown above the coefficient of x is negative and their graphs have a maximum point.

With respect to **Graph 1** $-x^2 - x + 1 = 0$ has two real solutions (**Figure 3-21**).

With respect to **Graph 2** $-x^2 - 2x + 3 = 0$ has no real solutions (**Figure 3-22**).

With respect to **Graph 3** $-x^2 + 6x - 9 = 0$ has one real solution (**Figure 3-23**).

In Summary

For a quadratic function of the form $y = ax^2 + bx + c$

- If a is positive the graph has a minimum point.
- If a is negative the graph has a maximum point.
- Quadratic graphs cross the x -axis no more than twice.

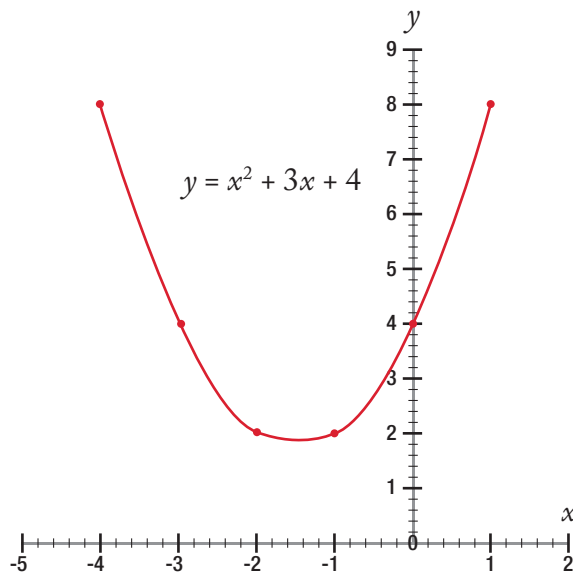


Figure 3-20. Graph of the function $y = x^2 + 3x + 4$.

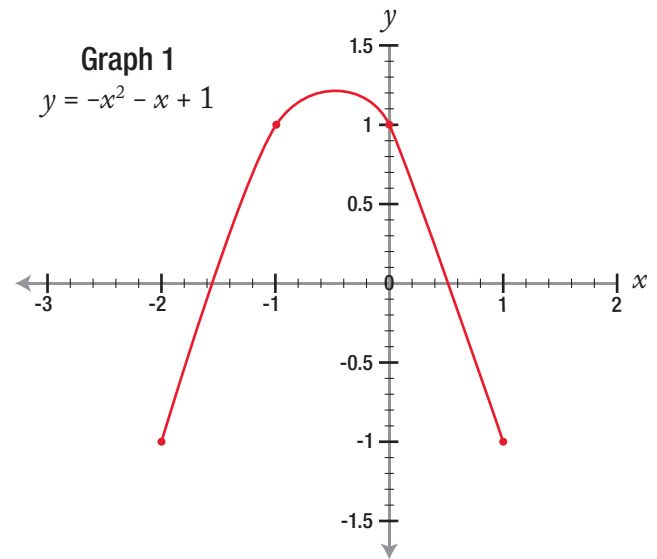


Figure 3-21. Graph of the equation $-x^2 - x + 1 = 0$.

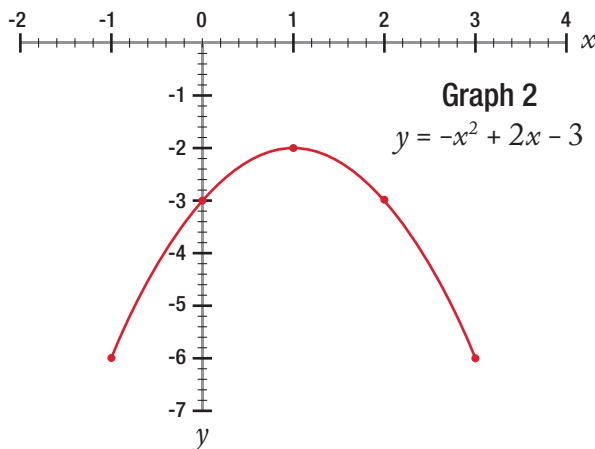


Figure 3-22. Graph of the equation $-x^2 - 2x + 3 = 0$.

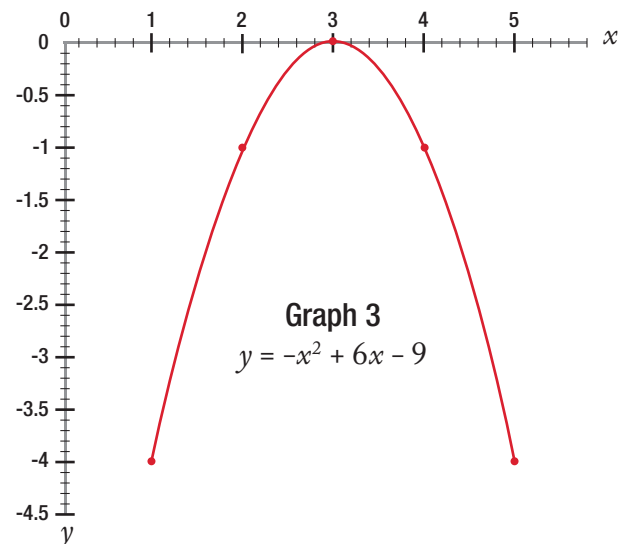


Figure 3-23. Graph of the equation $-x^2 + 6x - 9 = 0$.

TRIGONOMETRIC FUNCTIONS

Trigonometry is the study of the relationship between the angles and sides of a triangle. The word trigonometry comes from the Greek trigonon, which means three angles, and metro, which means measure.

RIGHT TRIANGLES, SIDES AND ANGLES

The triangle in **Figure 3-24** is a right triangle because angle C is a 90° angle. Notice that each angle is labeled with a capital letter. Across from each angle is a corresponding side, each labeled with a lower case letter. Side *a* is opposite from angle A, and is referred to as the *opposite side*. Side *b* is next to, or adjacent to, angle A and is therefore referred to as the *adjacent side*. Side *c* is always across from the right angle and is referred to as the *hypotenuse*.

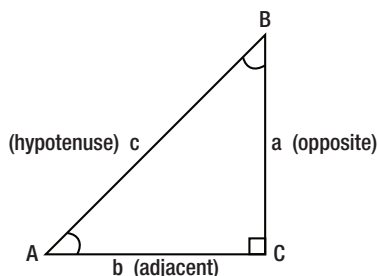


Figure 3-24. A right triangle with its corresponding sides and angles labeled.

TRIGONOMETRIC RELATIONSHIPS, SINE, COSINE AND TANGENT

Regardless of the lengths of the sides of a right triangle, the relationships between the angles and the sides of the triangle remain the same. The relationships or ratios between the sides are the same as long as the angles are the same. These ratios are known as functions. The three primary trigonometric functions and their abbreviations are: *sine* (*sin*), *cosine* (*cos*), and *tangent* (*tan*). The three trigonometric functions compare two of the sides of a right triangle as follows:

$$\text{Sine (sin) of angle A} = \frac{\text{opposite side (side a)}}{\text{hypotenuse (side c)}}$$

$$\text{Cosine (cos) of angle A} = \frac{\text{adjacent side (side b)}}{\text{hypotenuse (side c)}}$$

$$\text{Tangent (tan) of angle A} = \frac{\text{opposite side (side a)}}{\text{adjacent side (side b)}}$$

Refer to **Figure 3-21** to see these relationships of the sides as they pertain to angle A.

If the values of the angles are changed in a right triangle (except the 90° angle which must remain fixed for it to be a right triangle), the values for the functions sine, cosine and tangent will also change. For each unique value of angle A, there are unique values for the sine, cosine and tangent ratios for that angle. The values are the same regardless of the length of the sides of the triangle. They can be computed by actually measuring the sides of the given triangle or if the sides of the triangle are known. However, since the ratio does not change for any given angle value, and these ratios have been around for centuries, the sine, cosine and tangent function values have been calculated and recorded on tables such as the one in **Figure 3-25**.

These three functions can also be found on most scientific calculators. Typically, push the desired function key, enter the value of the angle in degrees and press enter. Remember, however that neither Tables nor Electronic Calculators are allowed in EASA examinations.

USING SINE, COSINE AND TANGENT TABLES

Common uses for sine, cosine and tangent are calculating an angle when the lengths of the sides are known or calculating the sides when the values of the angles are known. The following examples are given to instruct how to use trigonometric function tables and to illustrate possible uses for doing so.

In the **Figure 3-26**, the DME (*distance measuring equipment*) indication on an older aircraft measures and displays the distance from the aircraft directly to the ground based DME broadcast unit. This is known as the *slant distance* and is displayed as 13 nautical miles which is 24 076 meters. The aircraft altimeter indicates the aircraft is 3 765 meters above the ground.

A right angle is formed between the aircraft, the point directly under the aircraft and the DME ground broadcast station. Thus a right triangle exists between the ground station, the point under the aircraft and the aircraft itself.

ANGLE	SINE	COSINE	TANGENT	ANGLE	SINE	COSINE	TANGENT
0	0.000	1.000	0.000	23	0.391	0.921	0.424
1	0.017	1.000	0.017	24	0.407	0.914	0.445
2	0.035	0.999	0.035	25	0.423	0.906	0.466
3	0.052	0.999	0.052	26	0.438	0.899	0.488
4	0.070	0.998	0.070	27	0.454	0.891	0.510
5	0.087	0.996	0.087	28	0.469	0.883	0.532
6	0.105	0.995	0.105	29	0.485	0.875	0.554
7	0.122	0.993	0.123	30	0.500	0.866	0.577
8	0.139	0.990	0.141	31	0.515	0.857	0.601
9	0.156	0.988	0.158	32	0.530	0.848	0.625
10	0.174	0.985	0.176	33	0.545	0.839	0.649
11	0.191	0.982	0.194	34	0.559	0.829	0.675
12	0.208	0.978	0.213	35	0.574	0.819	0.700
13	0.225	0.974	0.231	36	0.588	0.809	0.727
14	0.242	0.970	0.249	37	0.602	0.799	0.754
15	0.259	0.966	0.268	38	0.616	0.788	0.781
16	0.276	0.961	0.287	39	0.629	0.777	0.810
17	0.292	0.956	0.306	40	0.643	0.766	0.839
18	0.309	0.951	0.325	41	0.656	0.755	0.869
19	0.326	0.946	0.344	42	0.669	0.743	0.900
20	0.342	0.940	0.364	43	0.682	0.731	0.933
21	0.358	0.934	0.384	44	0.695	0.719	0.966
22	0.375	0.927	0.404	45	0.707	0.707	1.000

Figure 3-25. Table of values for sine, cosine and tangent of right angles 0-45°. See reference for complete table of values.

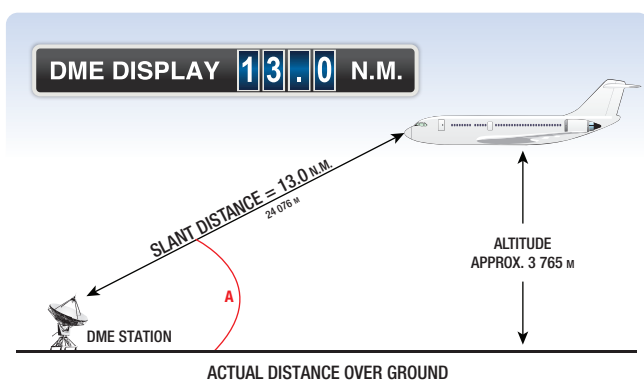


Figure 3-26. Using trigonometry to solve problems.

What is the distance over the ground from directly under the aircraft to the DME broadcast location? Since we now know that angle A is 90°, use the cosine function to find this unknown distance.

Cosine of 90° = the adjacent side/divided by the hypotenuse.

Find the cosign value of a 90° angle on the function chart, substitute values into the formula, and perform the arithmetic:

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$0.9877 = \text{adjacent side} / 24\,076\,m$$

$$0.9877 \times 24\,076\,m = \text{adjacent side}$$

$$23\,779.9\,m = \text{adjacent side}$$

23 779.9 meters is the distance from the DME broadcast station to the point directly under the aircraft.

Example:

For the triangle shown below find:

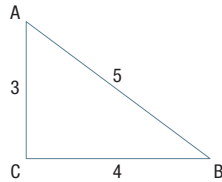
- the sine of angle A
- the tangent of angle A
- The cosine of angle B

Solution:

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{4}{5}$$

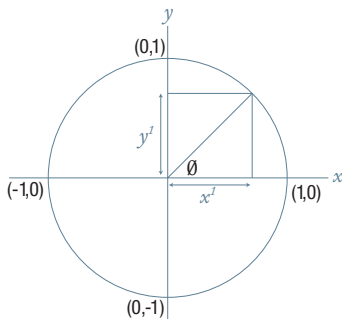
$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{4}{3}$$

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$$



TRIGONOMETRIC RATIOS FOR ANGLES GREATER THAN 90°

To find values of sine, cosine and tangent for angles greater than 90° we use the unit circle. A unit circle is a circle with a radius of length 1 unit.



Such a circle is shown in the diagram above. For the angle θ , we know that:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y_1}{1} = y_1$$

What this means is that the sine of an angle is a distance along the y -axis.

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x_1}{1} = x_1$$

What this means is that the cosine of an angle is a distance along the x -axis.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y_1}{x_1} = \frac{\sin \theta}{\cos \theta}$$

Using the unit circle above, it is easy to read off the sine, cosine and tangent values of angles of 0°, 90°, 180°, 270° and 360°. They are shown in the following table.

Cosine and Tangent Values					
Angle	0°	90°	180°	270°	360°
Sine	0	1	0	-1	0
Cosine	1	0	-1	0	1
Tangent	0	∞	0	∞	0

INVERSE TRIGONOMETRIC RATIOS

INVERSE SINE

Given $\sin \theta = x$ how do we find θ ?

We say $\theta = \sin^{-1}x$.

We read ' $\sin^{-1}x$ ' as sine inverse of x .

(This is sometimes called arcsin of x).

INVERSE COSINE

Given $\cos \theta = x$ how do we find θ ?

We say $\theta = \cos^{-1}x$.

We read ' $\cos^{-1}x$ ' as cosine inverse of x .

(This is sometimes called arccos of x).

INVERSE TANGENT

Given $\tan \theta = x$ how do we find θ ?

We say $\theta = \tan^{-1}x$.

We read ' $\tan^{-1}x$ ' as tan inverse of x .

(This is sometimes called arctan of x).

Example:

Given $\sin \theta = -1$ using the unit circle, we can write $\theta = \sin^{-1}(-1) = 270^\circ$.

PYTHAGORAS' THEOREM

Pythagoras' Theorem is named after the ancient Greek mathematician, Pythagoras (~500 B.C.). This theorem is used to find the third side of any right triangle when two sides are known. The Pythagoras' Theorem states that $a^2 + b^2 = c^2$. (**Figure 3-27**)

Where c = the hypotenuse of a right triangle, a is one side of the triangle and b is the other side of the triangle.

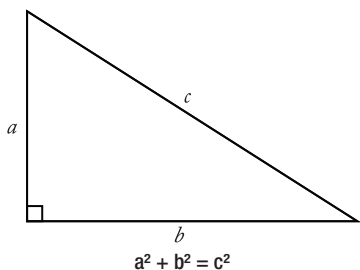


Figure 3-27. Pythagoras' Theorem is used to find the third side of a right triangle when the values of two sides are known.

Example:

What is the length of the longest side of a right triangle, given the other sides are 7 meters and 9 meters? The longest side of a right triangle is always side c , the hypotenuse. Use Pythagoras' Theorem to solve for the length of side c as follows:

$$a^2 + b^2 = c^2$$

$$7^2 + 9^2 = c^2$$

$$49 + 81 = c^2$$

$$130 = c^2$$

$$\text{If } c^2 = 130 \text{ then } c = \sqrt{130} = 11.4 \text{ meters}$$

$$\text{side } c = 11.4 \text{ meters}$$

Example:

The cargo door opening in a military airplane is a rectangle that is 1.75 meters tall by 2 meters wide. A section of square steel plate that is 2.25 meters wide by 2.25 meters tall by 2 centimeters thick must fit inside the airplane. Can the square section of steel plate fit through the cargo door?

It is obvious that the square steel plate will not fit horizontally through the cargo door. The steel plate is 2.25 meters wide and the cargo door is only 2 meters wide. However, if the steel plate is tilted diagonally, will it fit through the cargo door opening?

The diagonal distance across the cargo door opening can be calculated using the Pythagorean Theorem where " a " is the cargo door height, " b " is the cargo door width, and " c " is the diagonal distance across the cargo door opening.

$$a^2 + b^2 = c^2$$

$$(1.75 \text{ m})^2 + (2 \text{ m})^2 = (\text{diagonal opening})^2$$

$$3.0625 + 4 = 7.0625$$

$$\sqrt{7.0625} = 2.658 \text{ m}$$

Steel plate is 2.25 meters

The diagonal distance across the cargo door opening is 2.658 meters, so the 2.25 meters wide square steel plate will fit diagonally through the cargo door opening and into the airplane.

GRAPHS OF TRIGONOMETRIC FUNCTIONS

We graph trigonometric functions in exactly the same way as other functions i.e. we create a table of x values and calculate the corresponding y values. In this case, however, the x values will be angles.

Table of Values					
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0

Graph of $y = \sin x$ (Figure 3-28). This type of graph is a wave. It repeats itself every 2π so we say it has a period or wavelength of 2π . The amplitude, A , of a wave is the distance from the center line i.e. the x axis to the top of a crest or to the bottom of a trough.

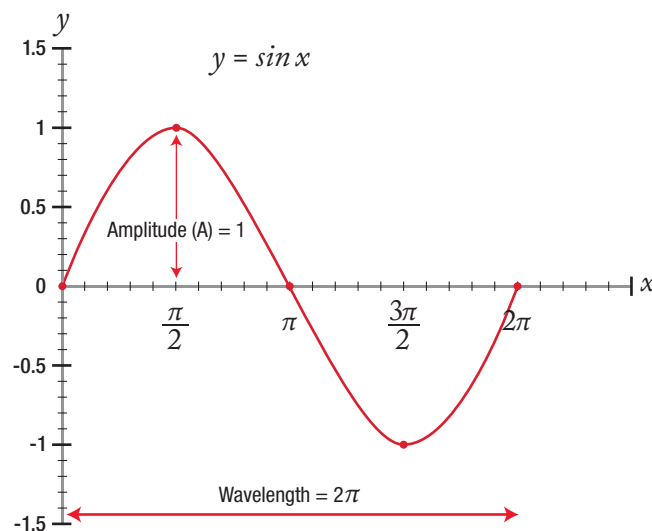


Figure 3-28. Graph of $y = \sin x$.

Figures 3-29 and 3-30, shown on the following page, illustrate the graphs of $y = 2\sin x$ and $y = 5\sin x$. You will notice the following:

- The graph of $y = \sin x$ has a maximum height of 1, and an amplitude of 1 (**Figure 3-28**).
- The graph of $y = 2\sin x$ has a maximum height of 2, and has an amplitude of 2 (**Figure 3-29**).
- The graph of $y = 5\sin x$ has a maximum height of 5, and has an amplitude of 5 (**Figure 3-30**).

Each of the three graphs above have wavelength 2π . **Figure 3-31** shows more examples of *polar coordinates*.

POLAR COORDINATES

Lines can be defined using their length, r , and the angle θ the line makes with the positive x -axis. These two values give a pair of coordinates locating the position of a point in a plane for the line shown in the diagram below. (**Figure 3-32**)

The line has 5 units along the x -axis and 7 units up the y -axis. We find the length of the line using Pythagoras' Theorem.

$$r^2 = 5^2 + 7^2$$

$$r = 8.6$$

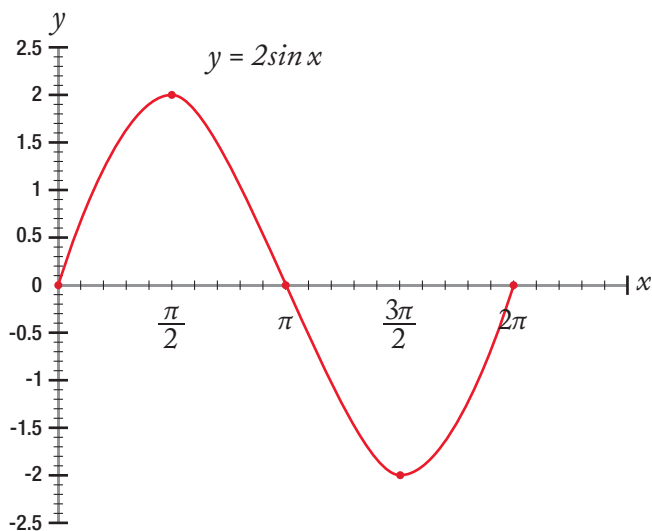


Figure 3-29. Graph of $y = 2\sin x$.

We find θ using the tangent function.

$$\tan \theta = \frac{7}{5}, \quad \theta = \tan^{-1}\left(\frac{7}{5}\right), \quad \theta = 54.46^\circ$$

The polar coordinates are $(8.6, 54.46^\circ)$.

The following graphs (**Figures 3-33 and 3-34**) show a few more graphs of polar coordinates.

The line shown above has length r given by:

$$r = \sqrt{3^2 + (-4)^2}$$

$$r = 5$$

The angle is given by:

$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

Then since the angle is in the fourth quadrant:

$$\theta = -53.13^\circ$$

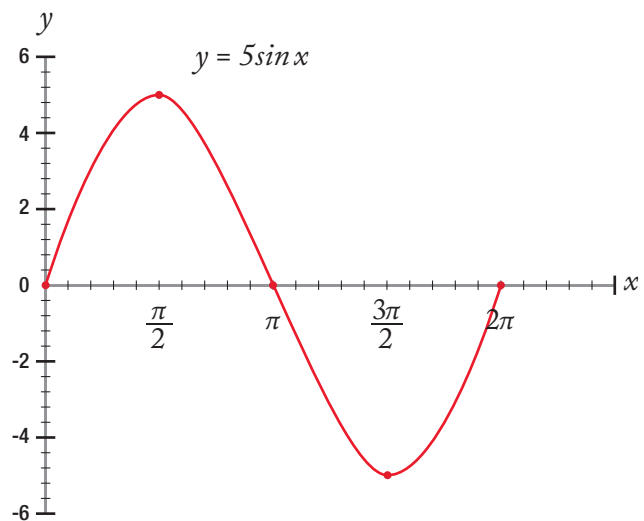


Figure 3-30. Graph of $y = 5\sin x$.

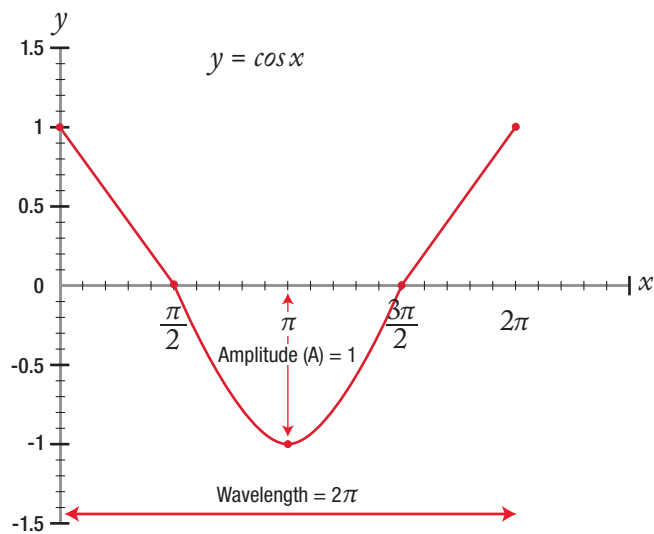


Figure 3-31. Graph of $y = \cos x$.

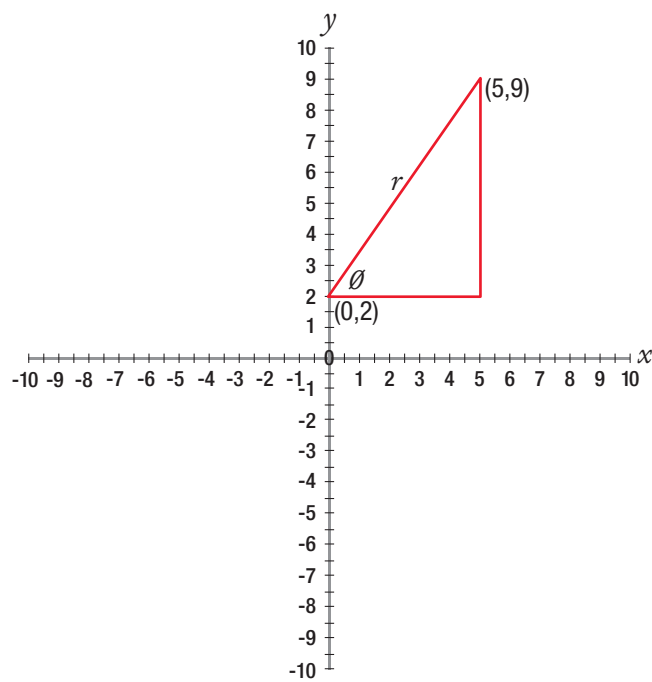


Figure 3-32. Polar Coordinates.

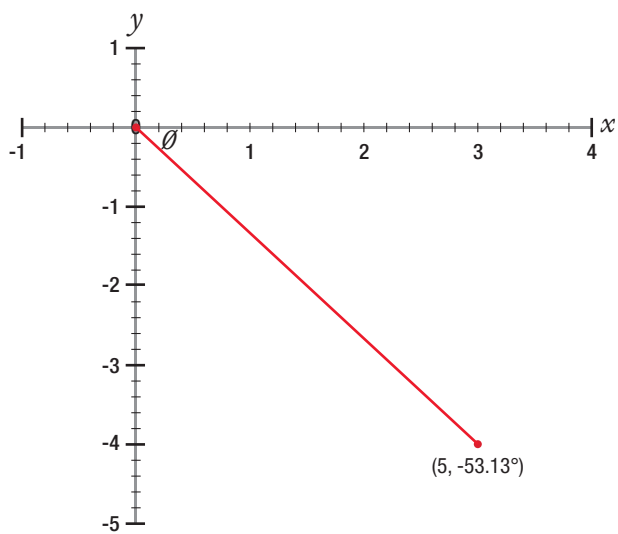


Figure 3-33. Polar Coordinates.

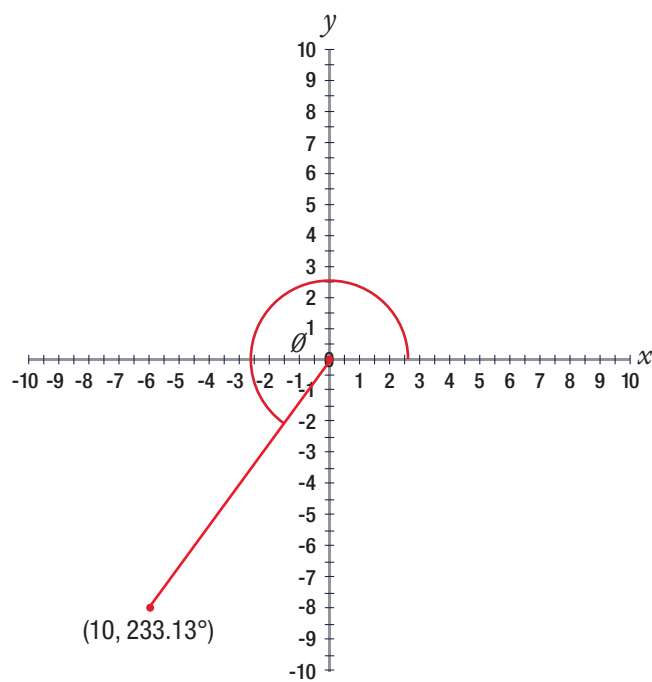


Figure 3-34. Polar Coordinates.

Question: 3-1

The graph of the function $y = 2x^2 - 2x - 4$ crosses the y-axis at?

- A. $y = 2, y = -2$
- B. $y = -4$
- C. $x = 1, x = -1$

Question: 3-2

The value of $\cos \pi$ is ?

- A. 1
- B. 0
- C. -1

Question: 3-3

Trigonometry is used to show the relationship between the sides and angles of rectangles.

- A. True
- B. False

Question: 3-4

Given $\sin A = \frac{3}{5}$ then $\tan A$ has the value?

- A. $\frac{4}{5}$
- B. $\frac{3}{4}$
- C. $\frac{5}{3}$

Question: 3-5

The lines $y = 3x + 5$ and $0 = 2y - 6x - 2$ are?

- A. Parallel
- B. Perpendicular
- C. Neither parallel or perpendicular

Question: 3-6

The graph of an equation illustrates?

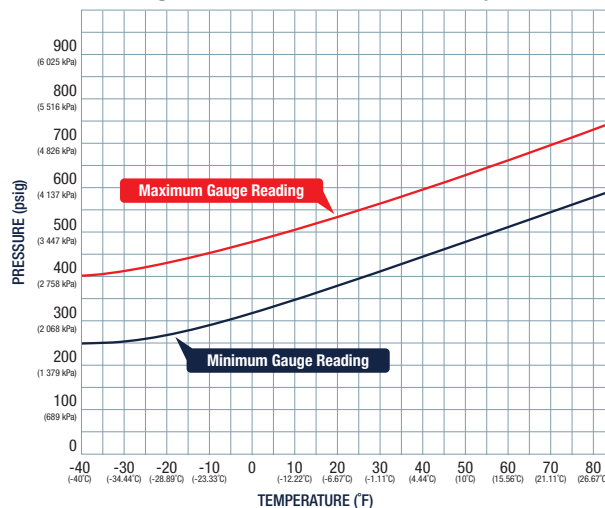
- A. The minimum values of the variables.
- B. The maximum value of the variables.
- C. All of the points where the equation is true.

Question: 3-7

On the following graph, what is the maximum pressure to which a fire extinguisher can be fill if the temperature is 40°F ?

- A. 449 psig
- B. 600 psig
- C. 400 psig

Fire Extinguisher Container Pressure - Temperature



ANSWERS

Answer: 3-1

Answer is B.

Where a graph crosses the y -axis x is zero.

Substituting $x = 0$ in to the function gives $y = -4$.

Answer: 3-5

Answer is A.

Writing both equations in the form $y = mx + c$ gives $y = 3x + 5$ and $y = 3x - 1$ and it can be seen that the slopes are equal.

Answer: 3-2

Answer is C.

Using the unit circle $\cos \pi = -1$.

Answer: 3-6

Answer is C.

There are often many solutions to an algebraic equation. When graphed, the line on the graph represents all of the points where the equation is true.

Answer: 3-3

Answer is A.

Trigonometry is the branch of mathematics dealing with the relationships between the sides and angles of plane or spherical triangles, and their associated calculations.

Answer: 3-7

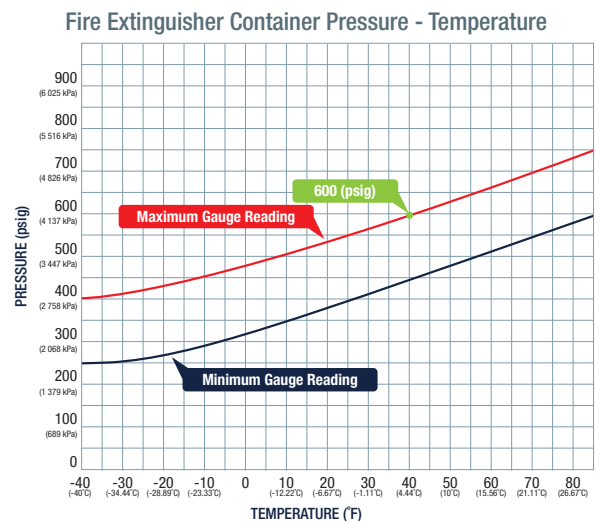
Answer is B.

Find 40° on the temperature scale. Go straight up until this line meets the maximum pressure (red) curve. From this intersection follow the graph directly to the left (*to the x -axis*) and read the pressure off the pressure scale (600 psig).

Answer: 3-4

Answer is B.

Draw a right angled triangle with sides 3, 4, 5.



Question: 3-8

The value of $\frac{\pi}{4}$ radians in degrees is

- A. 90°
- B. 60°
- C. 45°

Question: 3-10

The graph of $y = 4\sin x$ repeats itself every

- A. 4π
- B. π
- C. 2π

Question: 3-9

The line $y = -3x + 3$ crosses the x axis at the point

- A. (0, 3)
- B. (1, 0)
- C. (-3, 3)

Question: 3-11

For the graph of a quadratic function, which one of the following is always true?

- A. It crosses the x -axis at most twice.
- B. It crosses the y -axis exactly twice.
- C. It crosses the x -axis exactly twice.

ANSWERS

Answer: 3-8

Answer is C.

π radians = 180° – Therefore $\frac{\pi}{4}$ radians = 45°

Answer: 3-10

Answer is C.

A graph of $y = A \sin x$ repeats itself every 2π regardless of the value of A. The value of A is the amplitude of the wave.

Answer: 3-9

Answer is B.

Where a graph crosses the x -axis $y = 0$

Answer: 3-11

Answer is A.

The graph of any quadratic function has a maximum or a minimum value. The graph can turn above, below or on the x -axis. This means it crosses the x -axis at most twice.

COMMON MULTIPLIERS TO CONVERT TO SI

CONVERT FROM	TO	MULTIPLY BY
Distance		
inch (in)	meter (m)	0.025 4
foot (ft)	m	0.304 8
yard (yd)	m	0.914 4
mile (mi)	kilometer (km)	1.609
Velocity		
ft/s	m/s	0.304 8
mi/h or mph	km/h	1.609
mi/h or mph	m/s	0.447
Area		
in ²	square meters (m ²)	0.000 645
ft ²	m ²	0.092 9
mi ²	m ²	2 589 998
Volume		
in ³	cubic meter (m ³)	0.000 016 387
ft ³	m ³	2.831
quart (qt)	m ³	0.000 946
gallon (U.S. liquid) (gal)	m ³	0.004 546
Weight		
ounce (oz)	kilogram (kg)	0.028 35
pound (lb)	kg	0.453 6
ton (short 2 000 lbs)	kg	907.185
tonne	kg	1 000
Pressure		
pounds per square inch (psi)	kiloPascals (kPa)	6.897 757
pounds per square inch (psi)	Pascals (Pa)	6 894.757
Temperature		
°F (Fahrenheit)	°C	(t°F - 32) / 1.8
°C (Celsius)	°K (Kelvin)	t°C + 273.15
°K (Kelvin)	°C	t°K + 273.15

SI Multiplier Conversion Chart

REFERENCE

NUMBER	SQUARE	CUBE	SQUARE ROOT	CUBE ROOT	CIRCUMFERENCE	AREA
Number (N)	N Squared (N ²)	N Cubed (N ³)	Square Root of N (\sqrt{N})	Cube Root of N ($\sqrt[3]{N}$)	Circumference of a Circle with Diameter = N	Area of a Circle with Diameter = N
1	1	1	1.000	1.000	3.142	0.785
2	4	8	1.414	1.260	6.283	3.142
3	9	27	1.732	1.442	9.425	7.069
4	16	64	2.000	1.587	12.566	12.566
5	25	125	2.236	1.710	15.708	19.635
6	36	216	2.449	1.817	18.850	28.274
7	49	343	2.646	1.913	21.991	38.484
8	64	512	2.828	2.000	25.133	50.265
9	81	729	3.000	2.080	28.274	63.617
10	100	1 000	3.162	2.154	31.416	78.540
11	121	1 331	3.317	2.224	34.558	95.033
12	144	1 728	3.464	2.289	37.699	113.01
13	169	2 197	3.606	2.351	40.841	132.73
14	196	2 744	3.742	2.410	43.982	153.94
15	225	3 375	3.873	2.466	47.124	176.71
16	256	4 096	4.000	2.520	50.265	201.06
17	289	4 913	4.123	2.571	53.407	226.98
18	324	5 832	4.243	2.621	56.549	254.47
19	361	6 859	4.359	2.668	59.690	283.53
20	400	8 000	4.472	2.714	62.832	314.16
21	441	9 261	4.583	2.759	65.973	346.36
22	484	10 648	4.690	2.802	69.115	380.13
23	529	12 167	4.796	2.844	72.257	415.48
24	576	13 824	4.899	2.885	75.398	452.39
25	625	15 625	5.000	2.924	78.540	490.87
26	676	17 576	5.099	2.963	81.681	530.93
27	729	19 683	5.196	3.000	84.823	572.55
28	784	21 952	5.292	3.037	87.965	615.75
29	841	24 389	5.385	3.072	91.106	660.52
30	900	27 000	5.477	3.107	94.248	706.86
31	961	29 791	5.568	3.141	97.389	754.77
32	1 024	32 768	5.657	3.175	100.531	804.25
33	1 089	35 937	5.745	3.208	103.672	855.30
34	1 156	39 304	5.831	3.240	106.814	907.92
35	1 225	42 875	5.916	3.271	109.956	962.11
36	1 296	46 656	6.000	3.302	113.097	1 017.88
37	1 369	50 653	6.083	3.332	116.239	1 075.21
38	1 444	54 872	6.164	3.362	119.380	1 134.11
39	1 521	59 319	6.245	3.391	122.522	1 194.59
40	1 600	64 000	6.325	3.420	125.664	1 256.64
41	1 681	68 921	6.403	3.448	128.805	1 320.25
42	1 764	74 088	6.481	3.476	131.947	1 385.44
43	1 849	79 507	6.557	3.503	135.088	1 452.20
44	1 936	85 184	6.633	3.530	138.230	1 520.53
45	2 025	91 125	6.708	3.557	141.372	1 590.43
46	2 116	97 336	6.782	3.583	144.513	1 661.90
47	2 209	103 823	6.856	3.609	147.655	1 734.94
48	2 304	110 592	6.928	3.634	150.796	1 809.56
49	2 401	117 649	7.000	3.659	153.938	1 885.74
50	2 500	125 000	7.071	3.684	157.080	1 963.49

Functions of Numbers

NUMBER	SQUARE	CUBE	SQUARE ROOT	CUBE ROOT	CIRCUMFERENCE	AREA
Number (N)	N Squared (N^2)	N Cubed (N^3)	Square Root of N (\sqrt{N})	Cube Root of N ($\sqrt[3]{N}$)	Circumference of a Circle with Diameter = N	Area of a Circle with Diameter = N
51	2 601	132 651	7.141	3.708	160.221	2 042.82
52	2 704	140 608	7.211	3.733	163.363	2 123.71
53	2 809	148 877	7.280	3.756	166.504	2 206.18
54	2 916	157 464	7.348	3.780	169.646	2 290.22
55	3 025	166 375	7.416	3.803	172.787	2 375.83
56	3 136	175 616	7.483	3.826	175.929	2 463.01
57	3 249	185 193	7.550	3.849	179.071	2 551.76
58	3 364	195 112	7.616	3.871	182.212	2 642.08
59	3 481	205 379	7.681	3.893	185.354	2 733.97
60	3 600	216 000	7.746	3.915	188.495	2 827.43
61	3 721	226 981	7.810	3.937	191.637	2 922.46
62	3 844	238 328	7.874	3.958	194.779	3 019.07
63	3 969	250 047	7.937	3.979	197.920	3 117.24
64	4 096	262 144	8.000	4.000	201.062	3 216.99
65	4 225	274 625	8.062	4.021	204.203	3 318.30
66	4 356	287 496	8.124	4.041	207.345	3 421.19
67	4 489	300 763	8.185	4.062	210.487	3 525.65
68	4 624	314 432	8.246	4.082	213.628	3 631.68
69	4 761	328 509	8.307	4.102	216.770	3 739.28
70	4 900	343 000	8.367	4.121	219.911	3 848.45
71	5 041	357 911	8.426	4.141	223.053	3 959.19
72	5 184	373 248	8.485	4.160	226.194	4 071.50
73	5 329	389 017	8.544	4.179	229.336	4 185.38
74	5 476	405 224	8.602	4.198	232.478	4 300.84
75	5 625	421 875	8.660	4.217	235.619	4 417.86
76	5 776	438 976	8.718	4.236	238.761	4 536.46
77	5 929	456 533	8.775	4.254	241.902	4 656.62
78	6 084	474 552	8.832	4.273	245.044	4 778.36
79	6 241	493 039	8.888	4.291	248.186	4 901.67
80	6 400	512 000	8.944	4.309	251.327	5 026.54
81	6 561	531 441	9.000	4.327	254.469	5 152.99
82	6 724	551 368	9.055	4.344	257.610	5 281.01
83	6 889	571 787	9.110	4.362	260.752	5 410.60
84	7 056	592 704	9.165	4.380	263.894	5 541.76
85	7 225	614 125	9.220	4.397	267.035	5 674.50
86	7 396	636 056	9.274	4.414	270.177	5 808.80
87	7 569	658 503	9.327	4.431	273.318	5 944.67
88	7 744	681 472	9.381	4.448	276.460	6 082.12
89	7 921	704 969	9.434	4.465	279.602	6 221.13
90	8 100	729 000	9.487	4.481	282.743	6 361.72
91	8 281	753 571	9.539	4.498	285.885	6 503.88
92	8 464	778 688	9.592	4.514	289.026	6 647.60
93	8 649	804 357	9.644	4.531	292.168	6 792.90
94	8 836	830 584	9.695	4.547	295.309	6 939.77
95	9 025	857 375	9.747	4.563	298.451	7 088.21
96	9 216	884 736	9.798	4.579	301.593	7 238.22
97	9 409	912 673	9.849	4.595	304.734	7 389.81
98	9 604	941 192	9.900	4.610	307.876	7 542.96
99	9 801	970 299	9.950	4.626	311.017	7 697.68
100	10 000	1 000 000	10.000	4.642	314.159	7 853.98

Functions of Numbers (continued)

REFERENCE

ANGLE	SINE	COSINE	TANGENT	ANGLE	SINE	COSINE	TANGENT
0	0.000	1.000	0.000	46	0.719	0.695	1.036
1	0.017	1.000	0.017	47	0.731	0.682	1.072
2	0.035	0.999	0.035	48	0.743	0.669	1.111
3	0.052	0.999	0.052	49	0.755	0.656	1.150
4	0.070	0.998	0.070	50	0.766	0.643	1.192
5	0.087	0.996	0.087	51	0.777	0.629	1.235
6	0.105	0.995	0.105	52	0.788	0.616	1.280
7	0.122	0.993	0.123	53	0.799	0.602	1.327
8	0.139	0.990	0.141	54	0.809	0.588	1.376
9	0.156	0.988	0.158	55	0.819	0.574	1.428
10	0.174	0.985	0.176	56	0.829	0.559	1.483
11	0.191	0.982	0.194	57	0.839	0.545	1.540
12	0.208	0.978	0.213	58	0.848	0.530	1.600
13	0.225	0.974	0.231	59	0.857	0.515	1.664
14	0.242	0.970	0.249	60	0.866	0.500	1.732
15	0.259	0.966	0.268	61	0.875	0.485	1.804
16	0.276	0.961	0.287	62	0.883	0.469	1.881
17	0.292	0.956	0.306	63	0.891	0.454	1.963
18	0.309	0.951	0.325	64	0.899	0.438	2.050
19	0.326	0.946	0.344	65	0.906	0.423	2.145
20	0.342	0.940	0.364	66	0.914	0.407	2.246
21	0.358	0.934	0.384	67	0.921	0.391	2.356
22	0.375	0.927	0.404	68	0.927	0.375	2.475
23	0.391	0.921	0.424	69	0.934	0.358	2.605
24	0.407	0.914	0.445	70	0.940	0.342	2.747
25	0.423	0.906	0.466	71	0.946	0.326	2.904
26	0.438	0.899	0.488	72	0.951	0.309	3.078
27	0.454	0.891	0.510	73	0.956	0.292	3.271
28	0.469	0.883	0.532	74	0.961	0.276	3.487
29	0.485	0.875	0.554	75	0.966	0.259	3.732
30	0.500	0.866	0.577	76	0.970	0.242	4.011
31	0.515	0.857	0.601	77	0.974	0.225	4.331
32	0.530	0.848	0.625	78	0.978	0.208	4.705
33	0.545	0.839	0.649	79	0.982	0.191	5.145
34	0.559	0.829	0.675	80	0.985	0.174	5.671
35	0.574	0.819	0.700	81	0.988	0.156	6.314
36	0.588	0.809	0.727	82	0.990	0.139	7.115
37	0.602	0.799	0.754	83	0.993	0.122	8.144
38	0.616	0.788	0.781	84	0.995	0.105	9.514
39	0.629	0.777	0.810	85	0.996	0.087	11.430
40	0.643	0.766	0.839	86	0.998	0.070	14.301
41	0.656	0.755	0.869	87	0.999	0.052	19.081
42	0.669	0.743	0.900	88	0.999	0.035	28.636
43	0.682	0.731	0.933	89	1.000	0.017	57.290
44	0.695	0.719	0.966	90	1.000	0.000	∞
45	0.707	0.707	1.000				

Table of values for sine, cosine and tangent of right angles 0-90°.

fraction	decimal	mm	fraction	decimal	mm
1/64	0.015	0.396	33/64	0.515	13.096
1/32	0.031	0.793	17/32	0.531	13.493
3/64	0.046	1.190	35/64	0.546	13.890
1/16	0.062	1.587	39/64	0.562	14.287
5/64	0.078	1.984	37/64	0.578	14.684
3/32	0.093	2.381	19/32	0.593	15.081
7/64	0.109	2.778	39/64	0.609	15.478
1/8	0.125	3.175	5/8	0.625	15.875
9/64	0.140	3.571	41/64	0.640	16.271
5/32	0.156	3.968	21/32	0.656	16.668
11/64	0.171	4.365	43/64	0.671	17.065
5/16	0.187	4.762	11/16	0.687	17.462
13/64	0.203	5.159	45/64	0.703	17.859
7/32	0.218	5.556	23/32	0.718	18.256
15/64	0.234	5.953	47/64	0.734	18.653
1/4	0.25	6.35	3/4	0.75	19.05
17/64	0.265	6.746	49/64	0.765	19.446
9/32	0.281	7.143	25/32	0.781	19.843
19/64	0.296	7.540	51/64	0.796	20.240
5/16	0.312	7.937	13/16	0.812	20.637
21/64	0.328	8.334	53/64	0.828	21.034
11/32	0.343	8.731	27/32	0.843	21.431
23/64	0.359	9.128	55/64	0.859	21.828
3/8	0.375	9.525	7/8	0.875	22.225
25/64	0.390	9.921	57/64	0.890	22.621
13/32	0.406	10.318	29/32	0.906	23.018
27/64	0.421	10.715	59/64	0.921	23.415
7/16	0.437	11.112	15/16	0.937	23.812
29/64	0.453	11.509	61/64	0.953	24.209
15/32	0.468	11.906	31/32	0.968	24.606
31/64	0.484	12.303	63/64	0.984	25.003
1/2	0.5	12.7	1	1	25.4

Fractions to Decimal to Millimeter Conversions

OBJECT	AREA	FORMULA
Rectangle	Length × Width	$A = LW$
Square	Length × Width	$A = LW$
	Side × Side	$A = S^2$
Triangle	$\frac{1}{2} \times (\text{Base} \times \text{Height})$	$A = \frac{1}{2} BH$
Parallelogram	Length × Height	$A = LH$
Trapezoid	$\frac{1}{2} \times (\text{base}_1 + \text{base}_2) \times \text{Height}$	$A = \frac{1}{2} (b_1 + b_2)H$
Circle	$\pi \times \text{radius}^2$	$A = \pi r^2$
Ellipse	$\pi \times \text{semi-axis } a \times \text{semi-axis } b$	$A = \pi ab$

OBJECT	VOLUME
Rectangular Solid	LWH
Cube	S^3
Cylinder	$\pi r^2 H$
Sphere	$\frac{4}{3} \pi r^3$
Cone	$\frac{1}{3} \pi r^2 H$

Area Formulas

BINARY PLACE VALUE CHART

2^7 = 128	2^6 = 64	2^5 = 32	2^4 = 16	2^3 = 8	2^2 = 4	2^1 = 2	2^0 = 1
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Derivation of the place values of the binary number system.

DECIMAL PLACE VALUE CHART

10^7 = 10 000 000	10^6 = 1 000 000	10^5 = 100 000	10^4 = 10 000	10^3 = 1 000	10^2 = 100	10^1 = 10	10^0 = 1
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Place values of the decimal number system.

REFERENCE

MULTIPLICATION FACTOR	PREFIX	SYMBOL
1 000 000 000 000 000 000 = 10^{18}	exa	E
1 000 000 000 000 000 = 10^{15}	peta	P
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^9	giga	G
1 000 000 = 10^6	mega	M
1 000 = 10^3	kilo	k
100 = 10^2	hecto	h
10 = 10^1	deca	da
0.1 = 10^{-1}	deci	d
0.01 = 10^{-2}	centi	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p
0.000 000 000 000 001 = 10^{-15}	femto	f
0.000 000 000 000 000 001 = 10^{-18}	atto	a

SI Prefixes

COMMON CONVERSIONS

IMPERIAL SYSTEM		SI (METRIC)		SI (METRIC)		IMPERIAL SYSTEM
<i>Distance</i>				<i>Distance</i>		
1 Inch	is equal to	2.54 Centimeters		1 Centimeter	is equal to	0.394 Inches
1 Foot	is equal to	0.304 8 Meters		1 Meter	is equal to	3.28 Feet
1 (Statute) Mile	is equal to	1.609 3 Kilometers		1 Kilometer	is equal to	0.621 Miles
<i>Weight</i>				<i>Weight</i>		
1 Pound	is equal to	0.453 59 Kilograms		1 Kilogram	is equal to	2.204 Pounds
<i>Volume</i>				<i>Volume</i>		
1 Quart	is equal to	0.946 35 Liters		1 Liter	is equal to	1.057 Quarts
1 Gallon	is equal to	3.785 4 Liters		1 Lites	is equal to	0.264 Gallons
<i>Temperature</i>				<i>Temperature</i>		
0 Fahrenheit	is equal to	(-)17.778 Celsius		0 Celsius	is equal to	33.8 Fahrenheit
0 Fahrenheit	is equal to	255.37 Kelvin		0 Kelvin	is equal to	(-)437.87 Fahrenheit
<i>Area</i>				<i>Area</i>		
1 Square Inch	is equal to	6.451 6 Square Centimeters		1 Square Centimeter	is equal to	0.155 Square Inches
1 Square Foot	is equal to	0.092 903 Square Meters		1 Square Meter	is equal to	10.763 91 Square Feet
1 Square Mile	is equal to	2.59 Square Kilometers		1 Square Kilometer	is equal to	0.386 Square Miles
<i>Speed</i>				<i>Speed</i>		
1 Foot Per Second	is equal to	0.304 8 Meters Per Second		1 Meters Per Second	is equal to	3.281 Feet Per Second
1 Mile Per Hour	is equal to	1.609 3 Kilometers Per Hour		1 Kilometer Per Hour	is equal to	0.621 Miles Per Hour
1 Knot	is equal to	1.852 Kilometers Per Hour		1 Kilometer Per Hour	is equal to	0.540 Knots

Common conversions chart.

Addition. The process in which the value of one number is added to the value of another.

Algebra. The branch of mathematics that uses letters or symbols to represent numbers in formulas and equations.

Area. A measurement of the amount of surface inside a two-dimensional object.

Aspect ratio. The relationship of the length (*wingtip to wingtip*), or span, of an airfoil to its width, or chord.

Associative property. The sum or product of two or more terms is the same regardless of the order in which they are added or multiplied.

Base. In mathematics, used to refer to a particular mathematical object that is used as a building block. A base-*a* system is one that uses *a* as a new unit from which point counting starts again. (See *decimal system*.) In the mathematical expression a^n , read as "a to the nth power," *a* is the base.

Binary number system. The binary number system is a number system that has only two digits, 0 (*zero*) and 1. Binary numbers are made from a series of zeros and ones. An example of an 8-bit binary number is 11010010. The prefix "bi" in the word binary is a Latin root for the word "two."

Circle. A round plane figure whose boundary (the circumference) consists of points equidistant from a fixed point (the center).

Circumference (of a circle). The linear measurement of the distance around a circle. The circumference is calculated by multiplying the diameter of the circle by 3.1416.

Coefficient. In algebra, a number placed before and multiplying the variable in an algebraic expression (i.e., the 2 in $2x$).

Commutative property. The sum of term does not change regardless of the order in which the terms are added.

Compression ratio. The ratio of the volume of a cylinder with the piston at the bottom of its stroke to the volume of the cylinder with the piston at the top of its stroke.

Cone. A solid or hollow object that tapers from a circular base to a point.

Constant. In algebra, a number that appears on its own (without being combined with a variable).

Cosine (cos). A trigonometric function comparing two sides of a right triangle as follows:

$$\text{Cos} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

Cube. A symmetrical three-dimensional shape, either solid or hollow, contained by six equal squares.

Cylinder. A hollow or solid object with parallel sides the ends of which are identical circles.

D.M.E. Distance measuring equipment; an avionic device that indicates the distance of the aircraft to an input location.

Decibel (dB). The unit for measuring sound intensity. One decibel is the smallest change in sound intensity the human ear can detect.

Decimal system. The number system, also called the base-ten system, based on the number 10. Consisting of ten symbols, or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9), the main principle is that 10 is considered as a new unit from which point counting starts again.

Denominator. The lower part of a fraction (*represented by the letter D in N/D*), the quantity by which the numerator is divided.

Diameter (circle). The length of a line passing directly through the center of a circle. Twice the radius of the circle.

Difference. The answer to a subtraction problem.

GLOSSARY

Distributive property of multiplication. When multiplication is performed on two or more quantities already combined by another operation, the result is the same as when it is performed on each quantity individually and the products then combined.

Dividend. In a division problem, the number to be divided by the divisor. In $6 \div 2 = 3$, the dividend is 6.

Division. The process of finding how many times one number (*the divisor*) is contained in another number (*the dividend*).

Divisor. In a division problem, the number by which dividend is to be divided. In $6 \div 2 = 3$, the divisor is 2.

Ellipse. A regular oval shape, traced by a point moving in a plane so that the sum of its distances from two other points (the foci) is constant, or resulting when a cone is cut by an oblique plane that does not intersect the base.

Exponent (power). A shorthand method of indicating how many times a number, called the base, is multiplied by itself. For example, in the number 4^3 , the 3 is the power or exponent and 4 is the base. That is, 4^3 is equal to $4 \times 4 \times 4 = 64$.

Expression. In algebra, a collection of symbols that jointly express a quantity.

Fraction. A number written in the form N/D in which N is the numerator and D is the denominator. For example, $5/16$ is a fraction.

Function. A mathematic expression that defines the relationship between an input and an output.

Graph. A diagram showing the relation between variable quantities, typically of two variables, each measured along one of a pair of axes at right angles.

Hypotenuse. The side of a right triangle that is opposite the right angle. The hypotenuse is the longest side of a right triangle.

Improper fraction. A fraction with the numerator equal to or greater than the denominator.

Index: (plural: indices). The power or exponent of a number.

Logarithm. A quantity representing the power to which a fixed number (the base) must be raised to produce a given number.

Linear equation. An equation between two variables that gives a straight line when plotted on a graph.

Mixed number. A combination of a whole number and a fraction. For example, $5 \frac{3}{8}$ is a mixed number.

Multiplication. The process of repeated addition.

Negative number. A number that is less than zero.

Nomogram. A graph that usually consists of three sets of data. Knowledge of any two sets of data enables the reader to determine the third set.

Numerator. The upper part of a fraction (*represented by the letter N in N/D*).

Ohm. The standard unit used to measure resistance.

Ordered pair. A pair of variable such as (x, y) which describe the location of a point on a graph. The first variable (x) describes the distance from the y axis. The second variable describes the distance from the x axis.

Parallelogram. A four-sided plane rectilinear figure with opposite sides parallel.

Percentage. Used to express a number as a fraction of 100. Using the percentage sign, %, 90 percent is expressed as 90%.

Piston displacement. The total volume swept by all of the pistons of a reciprocating engine as they move during one revolution of the crankshaft.

Place value. The numerical value of a digit by virtue of its position in a number.

Polar coordinates. A pair of coordinates locating the position of a point in a plane, the first being the length of the straight line (r) connecting the point to the origin, and the second the angle (θ) made by this line with a fixed line such as the x axis.

Polynomial. An expression of more than two algebraic terms, esp. the sum of several terms that contain different powers of the same variable(s).

Positive number. A number that is greater than zero.

Power (exponent). A shorthand method of indicating how many times a number, called the base, is multiplied by itself. For example, in the number 4^3 , 3 is the power, or exponent, and 4 is the base. That is, 4^3 is equal to $4 \times 4 \times 4 = 64$.

Powers of ten. Also called scientific notation. It is a shorthand method of depicting very large or very small numbers.

Proportion. A proportion is a statement of equality between two or more ratios. The example of A is to B as C is to D can be represented $A:B = C:D$ or $A/B = C/D$.

Pythagoras' Theorem. An equation used to find the length of a third side of any right triangle when the lengths of two sides are known. The Pythagoras' Theorem states that $a^2 + b^2 = c^2$. The square of the hypotenuse (*side opposite the right angle*) is equal to the sum of the squares of the other two sides (a and b).

Quadratic equation. An equation in which the variable has a power of two.

Quotient. The result of dividing two numbers.

Radical sign. The symbol $\sqrt{\quad}$, used to indicate the root of a number.

Radius (circle). Equal to one-half the diameter of a circle.

Ratio. The comparison of two numbers or quantities.

Rectangle. A plane figure with four straight sides and four right angles, esp. one with unequal adjacent sides, in contrast to a square.

Rectangular coordinates. A pair of coordinates measured along axes at right angles to one another.

Root. A number that when multiplied by itself a specified number of times will produce a given number. The two most commonly used roots are the square root and the cube root.

Scientific notation. Used as a type of shorthand to express very large or very small numbers. For example, to express 1,250,000,000,000 in scientific notation is 1.25×10^{12} .

Signed numbers. A signed number can be either a positive or negative number. A positive number is a number that is greater than zero. A negative number is a number that is less than zero.

Simultaneous equation. Equations involving two or more variables that have the same value in each equation.

Sine (sin). A trigonometric function comparing two sides of a right triangle as follows:

$$\text{Sine} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

Square root. A non-negative number that must be multiplied by itself to equal a given number.

Subtraction. The process where the value of one number is taken from the value of another.

Slant distance. The direct distance from an aircraft to a input location on the surface of the earth.

Sphere. A round solid figure, or its surface, with every point on its surface equidistant from its center.

Square. A plane figure with four equal straight sides and four right angles.

Sum. The resulting answer in the addition process.

GLOSSARY

Tangent (*tan*). A trigonometric function comparing two sides of a right triangle as follows:

$$\text{Tan} = \frac{\text{adjacent side}}{\text{opposite side}}$$

Term. In algebra, each of the quantities in a mathematic expression.

Trapezoid. A four-sided figure with one pair of parallel sides.

Triangle. A three-sided figure in which the sum of the three angles equal 180° .

Trigonometry. The study of the relationships between the angles and sides of a triangle.

Variable. In algebra, a quantity represented by a letter or symbol that is assumed to be capable of varying.

Volume. The amount of space within a three-dimensional solid.

Whole numbers. The numbers: 0, 1, 2, 3, 4, 5, and so on.

A

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